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& Vosburgh, H. L.
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THESIS.

AN INVESTIGATION OF THE TENDENCIES IN CLASS-ROOM PRACTICES IN HIGH SCHOOL MATHEMATICS

(As determined by a study of College Board Examinations and tests given in High School.)

Submitted by

James Perkins Saunders
(S.B., M.I.T., 1930.)

and

Howard Leland Vosburgh
(A.B., Hamilton,1928.)

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First Reader: Edward J. Eaton, Professor of Education. Boston University
Second Reader: Jesse B. Davis, Professor of Education, Boston University

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I. INTRODUCTION.

A. Aim of the thesis---the problem.

Since the founding of the first universities and academies, mathematics has been accredited a position of honor and prestige among the subjects of college and high school curricula by educators and students alike. In fact, until recently, mathematics has been required of every student because of its assumed value to him.

The purpose of this thesis is to determine the past and present tendencies in the teaching of mathematics in the high schools of the United States; to compare these tendencies to see in what direction, if any, the trend in the teaching of mathematics is going; and to find out from a study of College Board Examinations and tests given in high schools if the present day practices are "in line", so to speak, with these tendencies.

B. The field of study and the limitations.

The field of investigation has been limited to Elementary Algebra and Plane Geometry---- to high schools only----and to New England. This limitation was necessary in order to restrict the size of the thesis problem.

C. The Methods of investigation and sources of material.

To determine the past and present tendencies a study has been made of the past and present literature upon the subject. The material has been obtained from books on the teaching of mathematics, reports of various committees, such as the National Committee on Mathematical Requirements, the Committee of Fifteen on the Geometry Syllabus, and other committees appointed at different times to study the teaching and trends of mathematics. The Yearbooks of the National Council of Mathematics Teachers have also been a source of material.

A study has been made of the College Entrance Board Examinations in Algebra and Plane Geometry from 1901 to the present to determine how the objectives may have changed.

In order to determine what the present tendencies are an analysis has been made of the final examinations given in representative high schools in New England in Elementary Algebra and Plane Geometry.

To obtain the tests as given in High Schools the following procedure was used:-

Sixty representative High Schools in New England were chosen according to size--some small, some large, and some of medium size--and letters,

addressed to the Head of the Mathematics Department, were sent requesting sample copies of final examinations as given in their high school in Algebra and Plane Geometry. From these letters approximately thirty replies were received, some enclosing tests in Algebra, some tests in Geometry, some tests in both subjects, and some no tests at all. The letters which enclosed no tests merely stated that the College Board or Regents examinations were used in the school for final examinations. Out of all these letters sufficient examinations were obtained to analyze sixteen schools in Algebra and seventeen in Geometry.

II. HISTORY OF AIMS, OBJECTIVES, AND TENDENCIES IN THE TEACHING OF SECONDARY SCHOOL MATHEMATICS.

A. Before 1900.

Geometry was first taught in the United States in the universities, and continued to be taught there until after the middle of the nineteenth century. Harvard College was founded in 1636, and arithmetic and geometry were taught there in the last year of the three years' course. One day a week was given to these studies together for three-quarters of the year. In 1655, Harvard adopted a four-year course but still taught mathematics in the last year. In 1726, printed textbooks first appeared, the first printed geometry used being that of John H. Alsted (1558-1638). About this time Euclid was first used at Harvard. Yale first used Euclid in 1733. Beginning in 1744, Yale taught geometry in the second year instead of in the last, and they continued to teach it in the second year until 1777. Not until 1788 did Harvard teach geometry in the second year, and they continued to teach it in the second year until 1818 when they changed to the first year. The University of Pennsylvania taught geometry in the first year along with arithmetic and algebra in 1758. At that time geometry consisted of the first six books of Euclid, books eleven and twelve, and plane and spherical trigonometry.*

In 1813, the "Analytical Society" was formed at Cambridge, England. This society aimed to encourage in Britain the rigorous study of French higher mathematics.

* A. W. Stamper - A History of the Teaching of Elementary Geometry - Chapter V.

The influence of this movement reached the United States so that in about ten years American writers began to adopt French texts. In 1819, John Ferrar, of Harvard, brought out a translation of Legendre's Geometry, which, with translations made by him of other French and Swiss texts on mathematics, were at once adopted in the leading American colleges. As a result of the widespread use of these geometries, Euclid was replaced by Legendre. The use of Euclid decreased until today the books of the English type are rarely used.*

One of the earliest geometries written by an American worthy of note was that of Benjamin Peirce. He favored the use of infinitesimals and also the use of the term direction, a concept probably first used in this country by Hayward, a Harvard teacher, in his geometry of 1829.

In 1851, Elias Loomis, of Yale, published a geometry which was revised in 1871. He had studied in Paris, and his book shows the French influence. It is claimed by some, that American writers, while they have given up Euclid, have so modified Legendre's geometry to make it look like Euclid's as much as possible.*

In 1871, Professor Olney, of the University of Michigan, published a geometry under two heads:**

I. Special or Elementary Geometry, comprising

1. Empirical Geometry.
2. Demonstrative Geometry.
3. Original Exercises.

* Final Report of the National Committee of Fifteen on the Geometry Syllabus - pages 29-32.

** Ibid. page 31.

4. Trigonometry.

II. General Geometry (Plane Loci).

Olney was a self-educated man and had many original ideas about teaching which forecast in many ways the present tendencies in mathematical teaching. His geometry shows that he attempted to correlate the various mathematical topics, and to introduce applications to everyday affairs.

Geometry occupied a small place in the Colonial Grammar Schools. The early grammar schools before the middle of the eighteenth century were not preparatory schools and the mathematics taught there "smacked of trade". Geometry was taught along with navigation and surveying. One of the oldest grammar schools with a different aim was the Boston Latin School, whose function was to prepare students for college. From 1815 to 1828, geometry was taught in the fourth and fifth years of the five-year course. The teaching seems to have been of high quality in this school.*

The development of the high school began at the end of the first quarter of the last century. The first high school was the English High School at Boston, founded in 1821. The first curriculum of the English High School comprised three years' work, and in the second year the program of studies included geometry and trigonometry with applications to the measurement of surfaces and solids. The nature of the mathematics taught indicates that the aim in view was for immediate practical service.*

* A. W. Stamper - A History of the Teaching of Elementary Geometry - Chapter V.

By 1844, when the universities began to place geometry on the list of entrance requirements, the high schools naturally took up more seriously the teaching of the subject. In 1871, when the University of Michigan inaugurated the system of accrediting schools, further stimulus was given to the teaching of geometry.*

Logical rigor in method was not required before the middle of the eighteenth century, judging from the textbooks in use at that time. Ward's mathematical books were in use at both Harvard and Yale, and his geometry begins with definitions, followed by twenty problems on constructions. These are followed by some propositions demonstrated in a loose fashion. The treatment of parallels in particular is not sound. As Ward's book was used at Yale as late as 1777, it is evident that Euclid's logic had no very strong hold on teaching. But during the last quarter of the eighteenth century, when the English influence became stronger, Euclid was taught more generally, and hence the teaching of geometry must have been characterized by greater logical rigor.*

The development of geometry teaching in the United States up to the last quarter of the nineteenth century may be summarized as follows: The universities first took up the teaching of geometry and it was not given over to the secondary schools until the middle of the last century. After 1821, when the first high school was established in Boston, geometry was placed in the secondary schools. The

* A. W. Stamper - A History of the Teaching of Elementary Geometry - Chapter V.

teaching in the early universities and in the schools was at first quite practical. Logic, in proofs, was stressed more when the English Euclids were in greatest use, during the last quarter of the eighteenth and first quarter of the nineteenth centuries. Notwithstanding the fact that the French influence, which began about 1817, has tended to make the teaching again more practical the English influence has been lasting. Except for a form of dogmatism that characterized some of the early teaching the demonstrative method has been in common use.

The researches in non-Euclidean geometry, which began in the eighteenth century in Italy and Germany, did not have any appreciable effect upon the teaching of geometry until the last quarter of the nineteenth century. The new ideas resulting from the researches in the non-Euclidean field have not affected the teaching of elementary geometry except in some of the definitions and postulates. They have assisted in the rejection of the definitions, "parallel lines are lines everywhere equally distant", and "parallel lines are straight lines which have the same direction". They have shown the futility of "proving" the parallel-postulate and have led to the use of the word "axiom", not as "self-evident truth", but as synonym for "postulate". **

** Report of the National Committee of Fifteen on the Geometry Syllabus - pages 31-32.

B. <u>1900 to the present</u>.

1. <u>Need for change</u>.

At the beginning of this century the need for a change both in the course of study and in the manner of presentation of Secondary Mathematics was very urgent. There was a considerable increase in the number of pupils in the mathematics classes. The great variations in ability to do the work had to be considered. Three decades before, pupils found in the classes of secondary schools had relatively high I.Q.'s. Only the brightest students in those days were able to survive to that stage of the educational system; and they were able to grasp the material as presented and gained much from the course.(1)

Algebra and Geometry <u>had to be made</u> less formal than it was half a century before. One could not expect to make the present classes enthusiastic merely over a logical sequence of proved propositions. It became necessary to make the work more concrete and to give a much larger number of simple exercises in order to create the interest and the satisfaction that comes from independent work, from a feeling of conquest, and from a desire to do something original.(1)

At the beginning of the present century the syllabi in mathematics in the high schools were determined largely by the requirements for entering our colleges. As

(1) Hassler and Smith - Teaching of Secondary Mathematics, Page 120, and Wm. D. Reeve - 4th Year Book, Pages 155-156, and Curran - The Place of Geometry in the Secondary Schools, Pages 27-28.

a rule examinations were set by each college for its candidates, the requirements being dictated by the department of mathematics.

2. Influences at work affecting changes.

a. Depleting Influences.

1. Tradition. Tradition is the largest factor that still hampers tendencies toward improvement. The fact that any topic has stood the test of time is some reason for respecting it, but it does not mean that age alone gives one topic precedence over another, nor that it should become sacred. The same applies to methods of instruction. In the past, subject matter has been emphasized instead of the development of mental maturity in students. The "subject" has become a habit with supervisors and teachers. It is regarded as something permanent and abiding while the student is transient.

2. Liberal Arts Colleges. Closely allied to tradition in retarding growth is the influence of the Liberal Arts College. If one is to judge from entrance requirements of the liberal arts college, to be a real "Liberal" one must worship at the shrine of the past. The higher the position he occupies in the fraternity the lower he kneels before the shrine of things old. (1)

3. Theory of Mental Discipline. Another influence which affected the aims of instruction a generation

(1) Frasier, Page 136, 4th Year Book, National Council of Math. Teachers.

ago was the theory of mental discipline. While this theory held sway, one type of algebra, for example, was thought to be as good as another for the education of youth. Now the general aim has shifted from the mental discipline point of view to that of giving the learner something that is practical. Still more significant is the modern viewpoint of stimulating and securing the interest of the pupil in his own activities and welfare. Not only the psychologists and laymen but also the teachers of mathematics have realized that this doctrine of mental discipline in teaching mathematics has been carried too far and has led to unfortunate conditions with reference both to content and material and to methods of teaching.

4. <u>Theory of Transfer Value</u>. It was formerly believed that mathematics should be studied because it engendered habits of logical thinking and precise work which would aid the pupil in other subjects studied, as well as establish certain valuable life habits. Except for the more gifted pupils this will not take place unless the subject is properly developed. The amount of "transfer" which takes place is very largely dependent upon methods of teaching. As the theory stands today, we can believe that transfer exists to the extent that identical elements are found in different situations, and that transfer is more readily possible in the case of attitudes and ideals[1]

(1) Teaching of Secondary Mathematics - Hassler and Smith Page 123.

than in specific skills. For this reason the modern movement in the reorganization of mathematics takes account of both disciplinary and utilitarian values.

b. Organizing Influences.

1. The Perry Movement. (1) Perhaps the first influence for a change toward better teaching was the Perry Movement in England. In 1901, John Perry made an address before the British Association which had far reaching results, not only in England but also in this country. His interest was in the education of the average citizen and was not in accord with the prevalent aim of the teaching of mathematics which was to pass examinations and to create mathematicians. He claimed that usefulness should determine what subjects should be taught and that new methods of teaching should be adopted so that the many who could not appreciate the usual type of mathematics could understand the subject and find use for it.

The details of his reform are familiar to mathematics teachers today, but in 1901 his ideas had not gained much headway even though his address was part of a movement that had been in existence for thirty years. Since Perry's address we have gone a long way in our progress.

2. Moore's address. The next epoch making (2) address was delivered by E.H. Moore, of the

(1) Teaching of Secondary Mathematics--Hassler and Smith Chapter VI.

(2) Ibid.

University of Chicago, before the American Mathematical Society, "On the Foundations of Mathematics". The second part of this address dealt with desirable changes in the teaching of elementary mathematics. Many of his ideas were similar to Perry's. He believed in less emphasis on the systematic and formal side and increased emphasis on the practical side. He also believed in the use of many postulates in geometry, leaving the more philosophical treatment until later.

3. The International Commission on the Teaching of Mathematics. An International Committee[1] on the teaching of mathematics was founded at Rome in 1908 at the suggestion of David Eugene Smith. This commission was appointed by the Fourth International Congress of Mathematicians to study the teaching of mathematics in the several countries. Reports were made in most of the leading countries showing the nature of the work done in schools of all types throughout the world. Those made in the United States were published by the Bureau of Education from 1911-1918. By means of these, teachers of mathematics in this country had an opportunity to compare their own work with the work being done in the other countries. It was found that after the fourth grade the schools of this country were poorer than the European schools. Since the time of this commission, much has been done in this country to remove these differences by change

(1) D.E. Smith - 1st Year Book - Pages 7 and 8.

in our junior high school courses of study. Due to the World War the influence of this commission was not nearly so great as it should have been, but it was far reaching as shown by the reforms made in the courses of study in our own country.

4. The College Board. The College Entrance(1) Examination Board was organized in 1900, and while this was a step forward, the influence of tradition still forced the retention of a great deal of material which is now considered obsolete and which the present College Entrance Board Syllabi removed.

College entrance requirements and examinations have always had an important effect upon the teaching of mathematics in secondary schools. Sometimes the influence has been ultraconservative and irksome. Too often have teachers concentrated their efforts on preparing pupils to pass these examinations, for their success is often judged by the results when the pupils take the examinations.

In 1922, a commission was appointed by the Board, which was made up of representatives from both secondary schools and colleges, to revise the entrance requirements. In 1923 this commission made a report in which it is evident that the interests of both the colleges and the secondary schools have been preserved. No longer can the criticism be made that the requirements and the examinations are made by those who know little or nothing

(1) Hassler and Smith - Teaching of Secondary Mathematics, Pages 109-110 and D.E. Smith - 1st Year Book, Pages 9-14.

about secondary school problems.

In Algebra the commission eliminated much useless material, such as long and elaborate manipulations with polynomials, and a great deal of the work in factoring. They gave more attention to the formula and the graph. The requirement in connection with fractions was lightened. The work with equations was limited to those which a pupil will use in subsequent study or use in every day life. The amount of time devoted to irrationals was shortened. Numerical trigonometry and logarithms were introduced.

In Geometry the number of book theorems was greatly reduced in order to place a greater emphasis upon original exercises. An examination covering the work of a single year in plane and solid geometry was introduced.

5. <u>The National Committee on Mathematical Requirements</u>. The National Committee on Mathematical[1] Requirements was organized in 1916 under the auspices of The Mathematical Association of America for the purpose of giving national expression to the movement for reform in the teaching of mathematics. The committee was first composed of six nationally known college professors of mathematics. Representatives of the secondary schools were added later at the request of the committee. It finally included six college professors and seven secondary school men. Tentative reports were published and discussed at

(1) Report of National Committee on Mathematical Requirements - Preface, and Wm. D. Reeve - 4th Year Book National Council, Page 145.

various meetings of organized groups of mathematics teachers throughout the country, criticism being invited. In this manner many progressive mathematics teachers in secondary schools have had a part in the work of the committee. In fact the committee states in the preface of its final report that its recommendations have the support of the great majority of the progressive teachers throughout the country. The improvement in the teaching of mathematics in the last decade has been very largely due to the work of this committee, not only on account of the value of its report, but also because of its preliminary work of organizing the teachers.

While the recommendations have had a great effect upon the curriculum and courses of study in both senior and junior high schools, they have had more influence in the junior high school. The reason for this is that the report was being made at the time that the courses of study for the junior high school were being planned.

The recommendations of the National Committee and of the College Board in Algebra and Plane Geometry are used as the basis of the study of the examinations in this thesis. They will be discussed further at that time.

c. Enriching Influences.

1. Correlated Mathematics.[1] In the early part of this century Professor Myers of the University of Chicago began in the University of Chicago High School a course in "Correlated Mathematics". This course placed great emphasis upon correlation and unification of mathematics in the high school. An attempt was made to eliminate the divisions between the mathematics subjects in the high school. Professor Breslich followed him, and he seems to regard the work before 1916 as the experimental part of the program leading to a well-established course.

The attempt to break down the teaching of mathematics in "water-tight" compartments resulted also in an attempt to correlate the work in mathematics with that of other sciences. The tendency to correlate various subjects like mathematics and physics has resulted in an attempt to bring about pure correlation within the subject itself.

2. General Mathematics. The movement for general mathematics in the secondary school dates back to about 1901, but owing to the influence of the colleges over the high school its progress was rather discouraging until the advent of the junior high school. In certain sections, where the junior high school is more popular, general mathematics in grades seven, eight, and nine has gained rapidly.

C. B. Walsh, in 1917, advocated a course of study which offered arithmetic and intuitive geometry in the

(1) Wm. D. Reeve - 4th Year Book, National Council of Mathematics Teachers - Page 143.

seventh grade; algebra in the eighth; demonstrative geometry, partly informal, in the ninth with elective work consisting of solid geometry, trigonometry, analytics, and calculus in the remaining three years.[(1)]

The vicious attacks on the "water-tight" compartments have been much more effective than seems noticeable at first sight. Many of the recent algebras and geometries because of the name "Algebra" or "Geometry" would thus seem to retain the "water-tight" system. However, in many of them algebra has found a place in the work in geometry and vice-versa.

3. <u>The Junior High School Movement</u>.[(2)] The junior high school movement began to be an important factor in American education about 1910. Every educator is now thoroughly familiar with the psychological foundations for this widespread movement to reorganize our system of education, making the elementary school cover the first six grades and the secondary school the next six. This movement created a situation in the seventh, eighth and ninth grades which gave mathematics teaching an opportunity for progress. Many textbook writers on junior high school mathematics organized the course in units and introduced considerable intuitive geometry and some algebra in the seventh and eighth grades. Most of them introduced trigonometry in the ninth grade, and some a unit of demonstrative geometry. The junior high school at its best is a revealing and

(1) Ibid, Page 144.

(2) Hassler and Smith - Teaching Secondary Mathematics, Pages 119-20.

exploring experience for the children, opening up large vistas in materials and situations which are helpful and meaningful. General mathematics has been introduced into most junior high schools, although the old four-year high schools have not accepted it.

This junior high school course of study is having its effect upon the senior high school. The tendency now in the tenth grade is to modify the idea that every proposition in geometry must be proved rigorously. Teachers now accept more facts intuitively and build upon these.(1)

4. Experimental Schools. The growth of experimental schools and the frequent reports in the "Mathematics Teacher" and in the year books of the National Council of Teachers of Mathematics of experimental work that is being carried on by teachers in public schools indicate the great interest in better teaching of mathematics material. Experiments with individual instruction, homogeneous grouping, laboratory instruction, large and small classes, and the like, indicate a professional interest on the part of teachers everywhere that is suggestive of a profressive trend.

The mathematics department of the University of Chicago High School, that of the Horace Mann School, and that of the Lincoln School of Teachers College, Columbia University, for example, have shown conclusively that it is possible to give their pupils an interesting and modern

(1) Reeve - 4th Year Book - Page 144.

course in mathematics and at the same time prepare them to pass college entrance examinations.[1]

5. Contribution of Psychology. The development of psychology from a speculative philosophy to an empirical science has affected both the content material and methods of instruction in our schools. The pupil, his capacities and his needs, have come in for an amount of attention never before accorded to him. The attempt is now to get the pupil's point of view. Although the greatest contribution has been made in the elementary field, the influence of advanced thinkers like Professor E. L. Thorndike has been felt all along the line. The spychologists have helped to organize the fundamental material along lines that are psychological rather than logical. As a result, subject matter has been made more concrete, content material has been organized in terms of the learner instead of the subject, and the entire atmosphere of the learning situation has been improved.[2]

6. Contribution of Sociology.[3] The educational sociologist has influenced the curriculum by bringing the social needs of the pupil more prominently before the teacher. The sociologist believes the individual should be adjusted vocationally, socially and ethically. His influence has tended to eliminate much useless and theoretical material from algebra and geometry and to substitute in its place useful and practical topics.

(1) Ibid, Page 147.
(2) Ibid, Pages 148-49.
(3) Joseph Jablonower - 7th Year Book - Page 3.

7. Schools of Education.[1] The achievements of schools of education have been manifested largely in the line of mental measurements which includes various types of tests. The results of these tests have been encouraging, although published tests have sometimes been used unwisely. However, schools of education have shown that mathematics can be adjusted to the capacities of young people, while the capacities of these pupils cannot be so readily adjusted to the old-style mathematics.

d. Materials of Instruction.

This term refers to topics and sources generally drawn upon to furnish the subject matter for the courses in mathematics.

1. Changes in the Course of Study. Traditionally materials have been placed in "logical" units- putting together subject matter which logically belongs under one topic. The pupil then concentrates upon one topic at a time without giving thought to its relation to other topics or to the course as a whole. To attain better results, it must be clearly understood that the organization must be made in "pedagogical" units rather than in "logical" units. This type of organization is best exemplified in the new mathematics of the junior high school.

a. Omissions and Additions, Elementary Algebra. Among the first topics to be eliminated

(1) D. E. Smith - 1st Year Book of National Council of Math. Teachers - Pages 5-6.

from elementary algebra were the highest common factor by division, cube root by the formula, the general theory of the quadratic, complicated brackets, complex fractions of a difficult type, simultaneous equations in more than three unknowns, the binominal theorem, and complicated radicals. The old idea that we must scientifically define all terms before they can be safely used and develop the subject logically has been replaced by a psychological development.

The graph began to receive the attention of mathematics teachers during the first decade of the present century. Teachers are beginning to realize the best approach to algebra is not by means of the equation or through the functions but through the study of the formula. The study of the graph is a major trend today in algebra because with the formula it helps to clarify the idea of functionality. The meaning of graphs can now be emphasized rather than the making of them.(1)

A significant trend in the teaching of algebra, not fully realized but well under way, is to put more meaning into the subject by replacing the emphasis upon formal symbolism by the function concept. The slowness with which this idea has been adopted in teaching probably accounts for the despair with which an occasional educator regards algebra.

As a reaction to the over-emphasis previously placed on manipulative skills in mathematics, stress is now put upon the importance of teaching the pupil not only to obtain

(1) E. G. Palmer - Hist. of the Graph in Elem. Alg. in the U.S. Sch. Sc. and Math. 12: 692-93.

the correct answer but also to think about, and understand, the meaning of the operations that he performs.

b. Omissions and Additions, Plane Geometry. Geometry, the oldest and most logical structure, has naturally resisted change more than any other subject in mathematics, and the changes are mostly on the surface. The texts are better, the amount of memory work has been diminished, and there has been more emphasis on original work. The fundamental change has been the introduction of the intuitive geometry in the seventh and eighth grades, and a short unit of demonstrative geometry in the ninth grade. This preliminary work has been one of the main factors in reducing the time spent upon the subject, so that one year of plane and solid geometry combined in the tenth grade is now thought by some authorities to be sufficient.(1)

Closely paralleling the trend in the simplification of algebra by omissions and additions of material came the suggestion that in geometry theorems whose meaning was already perfectly clear and obvious to the pupil should be postulated. The feeling is growing in some schools that the rigorous demonstration of such theorems should be either omitted entirely, or be deferred until the pupil's knowledge of geometry has advanced to a point where the logical implications of a proof have some significance for him.

Within the last few years the theory of limits has been omitted in the elementary courses. Even though this

(1) Allen, Gertrude - A Modified Program for School Geometry - Univ. High School Jour., 4:269-78.

topic is entirely omitted from most of the recent textbooks, it was commonly taught twenty years ago in most schools. In spite of very strenuous opposition, it occupies today a subordinate position in modern courses of study.(1)

There is at the present time an apparent desire to introduce a reasonable number of applied problems rather than to depend upon abstract propositions alone, and to arouse the interest of the pupil by an appeal to situations within his comprehension. It is even asserted by some that culture itself can be practical and taught in conjunction with things that are practical. In other words, it is not believed today that mathematics must be taught altogether as a pure science.

2. The Textbook. Among the various influences that have enriched and widened the teaching of mathematics is the progress that textbook writers have made. Professor David E. Smith has been one of the important factors in improving the general form of texts by his work in history and background of mathematics. This is seen in the use of pictures, better diagrams, in historical information, and in the reproduction of pages from interesting old books.

The makers of textbooks have cooperated in the movement to aid the progress of mathematics, by providing for the needs and interests of the children in matters of type, proper spacing, and the like.

(1) Lennes, N. J. - The Treatment of Limits in Elem. Geom.- School Sci. and Math., 6:52-58.

3. The Testing Program.[1] Additional evidence in the progressiveness of the teaching of mathematics lies in the fact that mathematicians have evolved and adopted certain standardized tests in this subject. These tests are valuable, for they are used not only to ascertain the achievement of the student and to discover just how much knowledge he has acquired but also to determine wherein his weaknesses, if any, lie. These latter kind are called diagnostic tests.

(1) Hassler and Smith - Teaching Secondary Mathematics - Pages 111-114.

III. CONSIDERATION OF THE PRESENT TENDENCIES IN PLANE GEOMETRY.

A. Summary of progress in the last quarter century.

Demonstrative geometry twenty-five years ago consisted of at least one year of plane geometry, following the course in algebra, and at least a half a year in solid geometry. In most schools there was a great deal of memorizing of demonstrations and the original exercise still played an almost negligible part, being, for many pupils, without either purpose of pleasure. A few teachers enlivened the work by applications of doubtful value, but on the whole it was generally looked upon as an intellectual grind.

The progress since that time has been steady and encouraging. Its nature may be summarized briefly as follows:*

1. There has been a more definite recognition by the schools that the chief purpose of demonstrative geometry is to show the application of logic to proof of mathematical statements. It therefore requires a maturity of mind hardly found before the tenth year of school, although for purposes of information a little work in demonstration may properly be given to pupils in the preceeding grade.

2. Therefore, the purpose of demonstrative geometry is not mensuration, this being sufficiently cared for in the work in intuitive geometry; its purpose, in part, is to demonstrate the truths already known intuitively. For this reason the work in the mensuration of the circle has little sanction in demonstrative geometry, the rules being known from intuitive geometry and the demonstrations as given not being very satisfactory from the standpoint of logic. The subject is therefore no longer required in college entrance examinations or for high school graduation. The same is true as to mensuration of the rectangle, the rectangular solid,

* D. E. Smith-Ist. Year Book, Nat. Council of Teachers of Mathematics-p. 27 ff.

the cylinder, the sphere, and the cone.

3. The number of demonstrated theorems, and especially of the corollaries, has been greatly reduced, the purpose being to retain only the basal propositions that are of most use in the demonstrations of the originals. This has shifted the emphasis from book proofs, which usually constituted all the geometry a century ago and most of that of the last quarter of the nineteenth century, to the original exercises where it belongs. Recent textbooks have an amount of original work of simpler character that was hardly imagined a generation ago.

4. The number of solved problems has been proportionately reduced quite as much as the number of demonstrated theorems. The simpler constructions with ruler and compass are given in intuitive geometry and their demonstrations are not of much value as compared with the demonstration of the theorems, leading as they do to only a small number of exercises and depending chiefly upon two or three simple theorems.

5. The exercises have greatly increased in number, but they have decreased in difficulty. The increase is due as already stated, to the shifting of emphasis from that which the author thought out for the pupil to that which the pupil is to think out for himself. The decrease in difficulty has arisen from the fact that the ability of pupils can certainly not be said to have increased during the period in which the schools have tended to the education of everyone rather than to that of a selected body of pupils of high intellectual promise.

There has, however, been another reason, the feeling that a large number of simple exercises trains the immature pupil better than a small number of difficult ones. In our efforts to conform to this belief we are still in the experimental stage. The pupil of mathematical inclinations will pre-

fer a more difficult type, and for him it will probably be better to pass rapidly over a few of the easy exercises and to come back to those requiring more thought as soon as possible.

6. The discussion and generalization of propositions now hold higher place than they did a score of years ago; at least it is rather more in evidence in our courses of study and in our textbooks.

7. There is a strong movement to cover the essentials of plane and solid geometry in a single year. This is often met by the assertion that it is impossible. If we eliminate the most of the construction problems, assume all the work in inequalities, eliminate the mention of incommensurables as applied to line-segments and the circle, all the theory of proportion, the work in the mensuration of solids, and the rather purposeless treatment of the spherical triangles, we can readily form a very satisfactory course for a single year.

At present two types of geometry are recognized, the first type is commonly referred to as "intuitive" geometry and the second as "demonstrative" geometry. This investigation was confined to plane geometry commonly taught in the tenth year of school and so demonstrative geometry is the only one that is discussed in this paper.

B. <u>The present objectives of Plane Geometry</u>.

The present objective of plane geometry is stated very clearly by Reeve who says,* "It is to make clear to the pupil the meaning of demonstration, the meaning of mathematical precision, and the pleasure of discovering absolute truth. If demonstrative geometry is not taught in order to enable the pupil to have the satisfaction of proving something, to train him in deductive thinking, to give him power to prove his own

* Wm. Reeve- 5th. Year Book-Nat. Council Teachers of Math. p. 11-12.

statements, then it is not worth teaching at all. Someone may ask, if training in constructive thinking is the big objective, why not give a course in pure logic? The answer is that geometry furnishes appropriate figures to illustrate and apply the essential types of thinking, while pure logic does not."

Longley summarizes the reasons for studying plane geometry under four heads:* a) Logical exposition, b) Geometric facts and relationships, c) Mensuration formulas and methods, and d) Cultivation of space perception.

The objectives reported by the National Committee on Mathematical Requirements do not differ materially from those stated by Reeve and Longley. The committee states,** "The principal purposes of instruction in plane demonstrative geometry are; to exercise further the spatial imagination of t the student, to make him familiar with the great basal propositions and their applications, to develop understanding and appreciation of a deductive proof and the ability to use this method of reasoning where it is applicable, and to form habits of precise and succinct statement, of the logical organization of ideas and of logical memory."

C. <u>Comparison of the recommendations of the National Committee on Mathematical Requirements with the syllabus of the College Entrance Examination Board.</u>

The National Committee on Mathematical Requirements was appointed in 1916, but it was not until 1923 that it brought out its report. In this same year (1923) the College Entrance Examination Board published its new syllabus defining the new requirements in plane geometry. These two reports will be compared to determine whether or not they differ materially in their recommendations of topics and propositions to be omitted or included in the teaching of plane geometry.

* Longley- 5th. Year Book, Nat. Council of M. T. p. 31-32.
** Report of the National Committee on Reorganization-p.34-35.

First, the personnel of these two bodies will be compared to determine who served on these committees and their importance in the field of mathematics teaching.

Members of the National Committee.

A. R. Crathorne, Professor of Mathematics, University of Illinois.

C. N. Moore, Professor of Mathematics, University of Cincinnati.

E. H. Moore, Professor of Mathematics, University of Chicago.

David Eugene Smith, Columbia University.

H. W. Tyler, Professor of Mathematics, Massachusetts Institute of Technology.

J. W. Young, Professor of Mathematics, Dartmouth College.

W. F. Downey, English High School, Boston, representing the Association of Teachers of Mathematics in New England.

Vevia Blair, Horace Mann School, New York, representing the Association of Teachers of Mathematics in the Middle Atlantic States and Maryland.

J. A. Foberg, State Normal School, California and Pennsylvania, representing Central Association of Science and Mathematics Teachers.

A. C. Olney, Commissioner of Secondary Education, California.

Raleigh Schorling, Professor of Mathematics, University of Michigan.

P. H. Underwood, Ball High School, Galveston, Texas.

Eula A. Weeks, Cleveland High School, St. Louis, Missouri.

Members of the College Entrance Examination Board.

William F. Osgood, Professor of Mathematics, Harvard University.

Mr. Thomas L. Bramhall, High and Latin School, Cambridge, Mass.

Mr. Ralph Beatley, Horace Mann School, (now in Harvard) New York City.

Mr. John A. Forberg, Department of Public Instruction, Harrisburgh, Pennsylvania.

Dunham Jackson, Professor of Mathematics, University of Minnesota.

William.R. Lonlgley, Professor of Mathematics, Yale University.

Mr. Islay F. McCormick, Albany Academy, Albany, N. Y.

Helen A. Merrill, Wellesley College, Professor of Mathematics.

David Eugene Smith, Columbia University.

Harry W. Tyler, Professor of Mathematics, Massachusetts Institute of Technology.

Dr. Eula A. Weeks, Cleveland High School, St. Louis, Missouri.

A comparison of the personnel of these two bodies shows that all of them are men or women who have made names for themselves, either in the field of secondary mathematics or college mathematics teaching.

Four members of the National Committee on Reorganization also served on the College Entrance Examination Board, namely, Professors D. E. Smith, Harry W. Tyler, Dr. Eula A. Weeks, and Mr. Foberg.

The recommendations of the National Committee are as follows: *

The formal theory of limits and of incommensurable cases be omitted, but that ideas of limit and of incommen-

* Report of the National Committee- pages 49, 74-77.

surable magnitudes receive informal treatment.

"It is believed that a more frequent use of the idea of motion in the demonstration of theorems is desirable, both from the view of gaining greater insight and of saving time.

"Certain definitions and explanations for definite usage are desirable such as for the circle, polygon, area of the circle, solids, circumference, obtuse angle, right triangle, rectangle, axiom, postulate, etc. Such terms as isosceles triangle, rectangle, parallelogram and segment be made as general as possible.

"Such terms as antecedent and consequent, trapezium, scholium, subtend, homologous, sect, lemma, oblong, perigon, etc. are inconsequential. Such terms as angle bisector, angle sum, explement, transverse angles, etc. should be avoided.

"Certain symbols should be recognized as international in use such as $\perp$ for perpendicular, // for parallel, $\cong$ for congruent, $\angle$ for angle and others of like nature, but the use of some should be discouraged as $\stackrel{o}{=}$ for equal in degrees, SAS for two sides and the included angle, etc. In lettering figures capitals should be used at the vertices, small letters for sides opposite the vertices and small Greek letters for angles.

The new requirements of the College Entrance Examination Board* redistribute the emphasis upon various parts of the work in order to lighten the demands upon the candidate's memory and to give him increased opportunity for attention to the development of geometrical understanding.

"One third of the examination will be devoted to book propostions from the starred list (for list of propositions adopted by College Entrance Examination Board see Appendix B), except in the case of a few theorems, not starred, which the student may be expected to work out as an original. The re-

* College Entrance Board Syllabus in Plane Geom., 1923-pages 2-5.

mainder will be divided between easy originals which ought to be within the reach of the properly trained student, and originals which, though still not of excessive difficulty, are designed to test the powers of the better candidates.

D. Comparison of theorems recommended by the National Committee and by the College Entrance Examination Board.

The National Committee appointed a sub-committee which prepared a list of propositions which they believed basal in the study of geometry. They based their selection on two criteria: 1) the extent to which the propositions and corollaries were used in subsequent proofs of important propositions and exercises; and, 2) the value of propositions in completing important pieces of theory.*

The College Entrance Examination Board selected eighty-nine theorems and twenty constructions which they believed important in the study of geometry.

Table I page 31 gives first the number of the proposition in the College Board Syllabus and then the number of the same proposition in the National Committee list of propositions. Table II page 32 gives the number of the proposition in the National Committee list and then the number of the same proposition in the College Board Syllabus.

In this manner we are able to see at a glance what propositions on the College Board list have been omitted by the National Committee and what propositions on the National Committee list have been omitted by the College Entrance Examination Board.

Note: Appendix A gives the list of propostions and constructions of the National Committee on Mathematical Requirements.

Appendix B gives the list of propositions and constructions of the College Entrance Examination Board.

* Report of National Committee- page 78.

Table I*

C. B. T. Number	Nat. Comm. Number of same prop.	C.B.T.	Nat. Comm.	C.B.T.	Nat. Comm.
1*	II A 1a	42	II A 24b	84	--
2*	1b	43	25	85	--
3* 3*	3	44	25	86	--
4*	3	45	III 13	87	II A 30
5*	1c	46	16	88	31
6*	I 5	47*.	II A 27	89	--
7	13	48	28		
8	II 3	49	III 15	Constructions	
9	-	50	17	1	II B 1
10*	II A 2	51	18	2	2
11*	6	52	--	3	3
12	III 2	53	14d	4	3
13	2	54	14d	5	4
14	II A 6	55	14c	6	5
15	III 1	56	14c	7	7
16	1	57	II A 12a	8	6a
17	4	58*	12b	9	6b
18	7a	60*	13b	10	6c
18	7b	59*	13a	11	9
19	7c	61*	13c	12	10
20*	II A 9a	62	17	13	13
21*	9b	63	17	14	13
22*	10	64	12c	15	--
23*	7	65	19	16	12
24	III 5	66*	20	17	13
25	6	67	14	18	14
26	14a	68	III 20	19	16
27	14a	69	II A 15	20	17
28	14b	70*	16		
29	14b	71	16		
30*	II A 4	72*	11a		
31*	5	73	11b		
32	III 11	74	III 21		
33	12	75*	II A 11c		
34	10	76*	18		
35	9	77*	18		
36*	-	78*	--		
37	II A 21	79	--		
38	22	80	II A 29		
39	23	81*	11d		
40*	II A 26	82	--		
41*	24a	83	III 22		

* College Board Theorems- Appendix B.
National Committee Theorems- Appendix A.

Table II*

Nat. Comm. number	Col. Ed. no. of same prop.
I	
1	-
2	-
3	-
4	6*
5	-
6	-
7	-
8	-
9	-
10	-
11	-
12	-
13	7
14	-
15	-
16	-
II A	
1a	1*
1b	2*
1c	5*
2	10*
3	3*
3	4*
4	30*
5	31*
6	11*,14*
7	23
8	--
9a	20*
9b	21*
10	22
11a	72*
11b	73
11c	75
11d	81
12a	57
12b	58*
12c	64
13a	59
13b	60*
13c	61

Nat. Comm.	C. B. No.
II A con't.	
14	67*
15	69
16	71,70
17	62,63
18	76*,77*
19	65,
20	66*
21	37
22	38
23	39
24a	41
24b	42
25	43,44
26	40*
27	47
28	48
29	80
30	87
31	88
III	
1	15,16
2	12,13
3	8
4	17
5	24
6	25
7a	18
7b	18
7c	19
8a	-
8b	-
9	35
10	34
11	32
12	33
13	45
14a	26,27
14b	28,29
14c	55,56
15	49

Nat. Comm.	C. B. No.
III con't.	
16	46
17	50
18	51
19	52
20	68
21	74
22	83
Constructions	
II B	
1	1
2	2
3	3,4
4	5
5	6
6a	8
6b	9
6c	10
7	7
8	-
9	11
10	12
11	13,14
12	16
13	17
14	18
15	-
16	19
17	20

* National Committee Theorems- Appendix A.
College Entrance Board Theorems- Appendix B.

1. Summary of Tables I and II.

The College Entrance Examination Board includes the following propositions in their list of requirements which the National Committee omit from their list.

Propositions nine, fifty-two, seventy-nine, eighty-two, eighty-four, eighty-five, eighty-six, and eighty-nine* are included by the C. E. E. B. but are omitted by the National Committee. These are unstarred propositions.

Propositions thirty-six and seventy-eight* are included by the C. E. E. B. but are omitted by the National Committee. These are starred propositions.

Construction number fifteen on the C. E. E. B. list is the only one omitted by the National Committee.

Of the first sixteen propositions* listed by the National Committee the C. E. E. B. includes but two on their list. However, the National Committee feels that these sixteen theorems should be accepted without proof or as postulates. Since the C. E. E. B. did not include such a list of propositions, it is not strange that they should be omitted from the C. E. E. B. list.

In the list of fundamental theorems recommended by the National Committee which they advise be used in selecting theorems to be proved on college entrance examinations there is but one, namely, number eight, that the C. E. E. B. does not include on their list.

On the subsidary list of propositions of the National Committee there are but two which are not included on the C. E. E. B. list. They are numbers eight(a) and eight(b.) The College Entrance Examination Board propositions which correspond to this subsidary list of the National Committee are all unstarred propositions.

The National Committee include construction fifteen

* For C. E. E. B. list of propositions see appendix B

* For Nat. Comm. list of propositions see appendix A

on their list which the C. E. E. B. omits from their list of constructions.

Conclusion.

These two organizations, the National Committee on Reorganization and the College Entrance Examination Board, working independently and publishing their reports at about the same time do not differ materially in determining what propositions and constructions are of major importance in plane geometry for;

The College Entrance Examination Board stars but one proposition which the National Committee would have the pupil assume without proof.

The College Entrance Examination Board stars but two propositions which the National Committee omit altogether.

The National Committee includes but one proposition in their list of "Fundamental Theorems" which the College Entrance Examination Board omits altogether.

The National Committee includes but two propositions in their list of "Subsidary Theorems" which the College Entrance Examination Board omits altogether.

The College Entrance Examination Board places but six unstarred propositions on their list which the National Committee omits from their list.

The College Entrance Examination Board includes but one construction which the National Committee omits.

The National Committee includes but one construction which the College Entrance Examination Board omits.

A. Tables from College Board examinations.

In order to determine whether or not the recommendations of the College Entrance Examination Board were actually being carried out the past examinations of this body were analyzed.

The last syllabus of the College Entrance Examination Board took effect in 1924. In order to determine whether the trend in theorems and originals has changed throughout the years from 1901 to 1931 all examinations were analyzed according to the last syllabus.

The examinations are divided into three periods. The first from 1930-1924, the second from 1923-1912, and the third from 1911-1901. A division was made between 1924 and 1923 for it was at this time the new syllabus took effect. A division was made between 1912 and 1911 because in 1912 the syllabus made by the Committee of Fifteen was first used.

Each examination was analyzed in this manner: on the 1930 paper, for example, the student was asked to prove the theorem, "A circle can be circumscribed about any polygon." According to the C. E. E. B. syllabus this is proposition seventy-eight*. So opposite the number 78 of table Ia and under the year '30 a number "1" was made to show this proposition was asked. One original on this paper was "What is the ratio of the areas of two circles whose radii are a and b?" To solve this original the pupil would have to know and use proposition 89 so opposite 89 and below '30 a figure "1" was made in table IIa. In case the student used a proposition in the proof of an original which is not in the last syllabus it was checked according to the place it occupies in "Plane Geometry" by Mirick, Newell, and Harper.**

The year of the examination is written across the top of the table and the number of the College Entrance Board proposition at the left of the table.

* List of C. E. E. B. propositions with their numbers may be found in Appendix B.

** See Appendix C.

Table Ia

College Entrance Board Examinations from 1930-1924.

Proofs of these propositions were asked

College Board No.*	'30	'29	'28	'27	'26	'25	'24	total
10*	..	..	..	1	..	..	..	1
20*	..	1	..	..	.1	..	..	2
30*	..	..	1	..	..	..	..	1
31*	..	..	..	..	..	1	..	1
47*	..	..	..	..	..	..	1	1
59*	1	..	..	..	..	..	..	1
60*	..	..	..	..	1	..	..	1
66*	..	..	..	..	..	..	1	1
67*	..	1	..	..	..	..	..	1
70*	..	..	1	..	..	..	..	1
76*	..	..	..	1	..	..	..	1
78*	.1	..	..	..	..	..	..	1
81*	..	..	..	..	..	1	..	1
total	2	2	2	2	2	2	2	14

Proofs of these constructions were asked.

11	..	..	..	..	1	..	..	1
20	..	..	..	.1	..	..	..	1
total				1	1			2

* Appendix B.

Table Ib

College Entrance Board Examinations from 1923-1912.

Proofs of these propositions were asked.

College Board No.*	'23	'22	'21	'20	'19	'18	'17	'16	'15	'14	'13	'12	total
5*	..	..	..	..	..	..	1	..	..	..	1	..	2
10*	..	..	..	..	..	1	..	..	..	..	..	..	1
11*	..	..	1	..	..	..	..	..	..	..	..	..	1
17	..	..	..	..	..	..	..	..	1	..	..	..	1
19	..	..	..	1	..	..	..	..	..	..	..	..	1
23*	1	1	..	..	..	..	..	..	..	..	..	..	2
26	..	..	..	..	..	..	..	..	..	1	..	..	1
29	..	..	..	..	1	..	..	..	..	..	..	..	1
32	..	..	..	..	..	..	..	..	..	..	..	1	1
40*	1	..	..	..	..	..	..	..	..	..	..	..	1
43	..	..	..	..	1	..	..	..	..	..	..	..	1
47*	..	..	1	..	..	..	..	..	..	..	..	..	1
50	..	1	..	..	..	..	..	..	..	..	..	..	1
51	..	..	..	..	..	1	..	..	..	..	..	..	1
52	..	..	..	..	..	..	..	..	..	1	..	..	1
58*	1	..	..	..	..	..	..	..	..	..	..	..	1
61*	..	1	..	..	..	..	..	1	..	1	..	..	3
62	..	..	..	..	..	..	..	..	..	..	..	1	1
63	..	..	..	1	..	..	..	..	..	..	..	..	1
67*	..	..	..	..	..	..	1	..	..	..	..	..	1
68	..	..	..	..	1	..	..	..	1	..	1	..	3
72*	..	..	..	..	..	..	..	1	..	..	..	..	1
75*	..	..	..	1	..	..	..	..	..	..	..	..	1
76	..	..	1	..	..	..	..	..	..	..	..	..	1
77*	..	..	..	..	..	..	..	..	1	..	..	..	1
81*	..	..	..	..	..	1	..	..	..	..	..	..	1
83	..	..	..	..	1	..	..	..	..	..	..	..	1
Art.** 189	..	..	..	..	..	..	..	..	..	..	..	1	1
Art. 233	..	..	..	..	..	..	..	..	..	..	1	..	1
totals	3	3	3	3	4	3	2	2	3	3	3	3	35

Proofs of these constructions were asked.

17	..	..	..	..	..	..	1	..	..	..	..	..	1

* Appendix B.
** Appendix C.

Table Ic

College Entrance Board Examinations from 1911-1901.

Proofs of these propositions were asked.

College Board No.*	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02	'01	
1*	..	..	..	..	..	..	..	..	..	..	1	1
2*	..	..	..	..	..	..	..	..	..	1	1	2
5*	..	..	..	..	1	..	..	..	..	..	1	2
18	..	..	1	..	..	..	..	..	..	..	..	1
20*	..	..	..	..	..	..	..	..	..	1	..	1
23*	..	..	..	1	..	..	..	..	..	..	..	1
25	..	..	..	..	..	..	..	..	..	..	1	1
27	..	..	..	..	..	..	1	..	..	..	..	1
30*	..	..	..	..	1	..	..	..	..	..	..	1
32	1	..	..	..	..	..	..	..	..	..	..	1
34	..	..	1	..	..	..	..	..	..	..	..	1
35	..	1	..	..	..	..	..	1	..	..	..	2
40*	..	..	..	..	..	..	..	..	1	..	..	1
43	..	..	..	..	..	..	..	..	..	1	..	1
46	..	..	..	..	..	..	..	1	..	..	..	1
47*	..	..	..	..	..	..	..	..	..	1	..	1
50	..	..	..	..	..	..	..	..	1	..	..	1
51	..	..	..	..	1	..	..	..	..	..	..	1
55	..	..	1	..	..	..	..	..	..	..	..	1
57	..	..	..	..	..	..	..	..	..	.1	..	1
59*	..	..	..	..	..	..	..	..	..	..	1	1
62	..	..	..	1	..	..	..	..	..	1	..	2
66*	1	1	..	..	..	..	..	..	..	1	..	3
68	1	..	1	..	1	..	..	..	..	..	1	4
72*	..	..	..	..	..	..	..	..	....	.1	..	1
73	..	..	..	..	..	..	..	..	..	..	1	1
75*	..	..	..	..	1	..	..	..	..	..	..	1
76*	1	..	..	..	..	..	..	..	..	1	..	2
77*	..	..	..	..	..	..	..	1	..	..	..	1
78*	..	..	..	..	..	..	..	..	..	..	1	1
79	..	..	..	..	..	..	..	..	..	..	1	1
82	..	..	..	..	..	..	..	..	..	1	..	1
86	..	..	..	..	..	..	.1	..	..	..	..	1
87	..	..	..	..	..	..	..	..	..	.1	1	2
Art. 232**	..	..	..	..	..	..	1	1	..	1	1	4
Art. 133	..	1	..	1	..	1	..	..	..	1	..	4
Art. 113	..	..	..	..	..	1	..	1	1	..	..	3
Art. 230	..	..	..	..	..	..	..	..	..	..	1	1
total	4	3	4	3	5	2	3	5	3	13	12	57

* Appendix B.

** Appendix C.

Table IIa

College Entrance Board Examinations from 1930-1924.

This table shows what theorems were used in proving originals given in these years and how often each was used on each paper.

College Board No.*	'30	'29	'28	'27	'26	'25	'24		total
1*	..	1	..	..	..	..	1		1
2*	..	..	..	..	3	..	..		3
3*	..	3	..	..	..	..	1		4
4*	..	..	..	1	..	..	..		1
10*	1	..	2	..	..	..	..		3
11*	1	..	..	1	..	..	1		3
12	1	..	..	..	..	..	..		1
15	..	..	..	1	..	..	..		1
18	1	..	..	1	..	..	1		3
23*	.1	1	..	2	..	1	..		5
24	..	2	..	..	..	1	1		4
31*	1	..	..	..	..	..	..		1
35	.1	..	..	..	..	..	..		1
37	1	..	..	..	..	..	..		1
38	..	1	..	..	..	..	..		1
40	..	..	1	..	..	..	..		1
57	..	..	..	..	..	1	..		1
59*	..	..	..	..	..	1	..		1
66*	5	1	1	2	1	2	1		13
68	..	..	..	..	.1	..	..		1
72*	1	..	1	..	..	..	..		2
73	5	1	..	2	..	..	..		8
76*	1	..	..	..	..	..	..	..	1
88	1	1	2	2	..	5	..		11
89	1	..	..	..	..	..	..		1
Art.** 106	..	..	1	..	..	..	1		2
Art. 199	1	1	..	2	..	..	2		6
total	23	12	8	14	5	11	9		82

Constructions Used.

3	..	..	..	1	..	..	..	1
5	..	..	..	1	..	..	..	1
6	..	..	..	1	..	..	..	1
7	1	..	..	..	..	..	..	1
17	1	..	..	..	..	..	..	1
total	2			3				5

* Appendix B.

** Appendix C.

Table IIb

College Entrance Board Examinations from 1924-1912.

This table shows what theorems were used in proving originals given in these years and how often each was used on each paper.

College Board No.*	'23	'22	'21	'20	'19	'18	'17	'16	'15	'14	'13	'12	total
1*	..	..	..	..	..	..	1	..	1	..	..	..	2
3*	..	..	..	..	..	..	..	..	..	..	..	1	1
4*	1	..	..	..	..	..	..	..	..	..	..	..	1
5*	..	..	..	..	..	..	..	1	..	..	..	..	1
10*	..	..	..	..	..	..1	..	..	..	..	..	..	1
11*	..	..	..	..	..	..	1	..	..	..	..	..	1
18	..	..	..	..	..	..	2	..	..	..	..	..	2
19	..	..	..	..	..	..	..	..	1	..	..	..	1
21*	..	..	..	..	..	..	1	..	..	..	..	..	1
23*	..	..	..	..	1	..	..	..	..	..	..	..	1
24	..	..	..	..	1	..	..	..	..	..	..	..	1
25	..	..	..	..	..	..	..	1	..	..	..	..	1
41*	..	..	..	1	..	1	..	..	..	..	..	..	2
44	1	..	..	..	..	1	..	..	..	..	..	..	2
47*	..	1	..	..	..	..	..	..	..	3	..	..	4
49	..	..	..	1	..	2	..	..	..	..	..	..	3
50	..	1	..	..	..	..	..	..	..	..	..	..	1
51	..	..	..	..	..	..	..	..	..	..	1	..	1
57	..	..	..	..	..	..	..	..	..	..	..	1	1
59*	..	1	...	..	..	1	..	..	..	1	..	..	3
60*	..	..	..	1	..	..	..	..	..	..	..	..	1
61*	..	1	1	..	..	..	..	..	..	..	..	..	2
66*	1	1	3	1	..	1	2	1	2	1	..	2	15
67*	..	..	..	..	..	..	1	..	..	..	..	..	1
68	..	..	..	..	..	..	..	..	..	..	1	..	1
70*	..	..	..	..	..	..	..	..	..	..	1	..	1
72*	..	..	..	..	..	..	..	1	1	..	..	..	2
73	..	1	..	..	..	..	..	1	..	2	..	..	4
76*	..	..	..	..	1	..	..	..	..	..	..	..	1
81*	..	..	..	..	..	..	..	1	..	..	..	..	1
86	1	..	..	..	..	..	..	..	..	..	..	..	1
87	2	..	..	1	1	..	2	1	2	1	..	..	10
88	..	1	..	..	..	..	2	..	..	1	1	..	5
Art.** 73	..	..	..	..	..	..	..	..	1	..	..	..	1
Art. 105	..	1	..	1	..	..	..	..	..	..	..	..	2
Art. 106	..	..	2	..	..	..	..	..	..	..	..	..	2
Art. 199	1	1	..	1	..	..	..	..	1	1	1	..	6
Art. 230	..	..	..	..	...	..	..	1	1	1	1	..	4
Art. 233	..	..	..	..	..	..	..	..	..	1	1	1	3
total	7	9	6	7	4	7	12	8	10	9	10	5	94

* Appendix B

** Appendix C

Table IIc
College Entrance Board Examinations from 1911-1901.

This table shows what theorems were used in proving originals given in these years and how often each was used on each paper.

College Board No.*	'11	'10	'09	'08	'07	'06	'05	'04	'03	'02	'01	Total
						years						
1*	..	..	..	..	..	..	..	1	..	..	..	1
2*	..	..	..	..	..	..	..	1	..	..	..	1
3*	..	..	2	..	..	..	..	1	..	..	..	3
4*	..	..	..	..	..	..	1	..	..	..	..	1
5*	..	..	..	..	..	1	..	..	..	..	..	1
18	..	..	..	..	..	..	..	..	1	..	..	1
23*	1	..	..	..	1	..	..	..	1	..	..	3
24	.1	..	..	..	..	..	..	..	..	..	..	1
25	..	..	..	1	..	2	..	..	2	..	..	5
37	..	..	..	..	..	..	..	1	..	..	..	1
39	..	..	..	..	..	..	1	1	..	..	..	2
40*	..	..	..	..	..	..	1	..	1	..	..	2
44	..	..	..	..	..	..	..	..	..	1	..	1
47*	..	..	..	..	..	1	2	2	..	..	..	5
49	..	..	..	..	..	..	..	1	..	..	..	1
50	..	..	..	..	..	..	..	1	.1	..	..	2
51	..	..	..	..	1	..	..	..	..	..	..	1
57	..	..	1	..	..	..	..	..	..	..	..	1
59*	..	..	1	..	..	..	..	1	..	..	..	2
60*	..	..	..	..	..	..	..	..	1	..	..	1
61*	1	..	..	..	1	..	..	1	1	..	..	4
66*	2	2	2	1	3	..	..	..	..	1	..	11
68	..	..	..	1	..	..	..	..	..	..	..	1
72*	..	..	..	..	..	..	..	..	1	..	2	3
73	1	..	1	..	..	..	..	..	1	..	..	3
75	..	..	1	..	1	..	..	..	..	..	..	2
77*	..	..	..	..	..	..	1	..	..	..	..	1
82	..	..	..	..	..	..	1	..	..	..	..	1
87	..	1	2	1	..	..	..	..	..	..	1	5
88	1	..	..	1	2	1	1	..	1	2	4	13
89	..	..	..	1	..	..	..	..	1	..	..	2
Art.** 105	1	..	..	..	1	..	..	..	..	..	..	2
Art. 199	1	..	..	1	..	..	..	..	..	2	2	6
Art. 230	..	..	..	..	..	1	1	..	..	1	2	5
Art. 310	..	..	..	..	..	1	..	..	..	..	..	1
total	7	5	10	7	10	7	9	11	12	7	11	96

* Appendix B
** Appendix C

Supplementary Table

The following was omitted from Table Ic page

Construction number 17 was called for in 1906 and in 1903. Construction number 20 in 1902.

The following was omitted from Table IIb page

Table IIb (con't)

Constructions

These constuctions were used in proving originals or doing exercises on these papers.

College Board No*	'23	'22	'21	'20	'19	'18	'17	'16	'15	'14	'13	'12	Total
1	..	..	..	2	.1	..	..	..	..	..	..	.1	4
2	..	..	..	..	..	..	..	1	..	..	..	1	2
4	..	..	..	..	1	2	..	1	..	..	1	..	5
5	..	..	..	..	..	..	..	1	1	..	..	.1	3
6	..	1	..	..	..	..	1	..	..	..	..	1	3
8	..	..	..	..	..	..	..	..	..	..	1	..	1
9	..	..	..	..	..	2	..	..	..	..	..	1	3
10	..	..	..	..	..	..	..	..	..	..	1	..	1
17	..	..	..	..	..	..	..	..	1	..	..	..	1
total		1		2	2	4	1	3	2		3	5	23

Table IIc

Constructions

Construction number eight and number 15 were used in the solution of exercises on the 1911 paper.

Table IId

This table indicates what definitions were asked for on the College Entrance Board Examination papers between 1930 and 1901.

In 1903, parallelogram and similiar figures.

In 1902, triangle, parallelogram, circle, tangent, similiar figures.

In 1901, angle, diagonal, axiom, postulate, plane and surface.

* Appendix B.

1. Summary.

From 1900 to 1924 the student was asked to prove the following theorems which are not found in the 1923 syllabus of the College Entrance Examination Board. Art.* 113 in 1906, in 1904, and in 1903; Art. 133 in 1910, in 1908, in 1906, and in 1902; Art. 230 in 1901; Art 232 in 1905; and Art. 233 in 1913.

Out of the 192 times that proofs were called for during those years, only 11 or 13% of the proofs would demand that the pupil be familiar with a proposition not found in the 1923 syllabus.

Fifty-five propositions were used in these 192 proofs, six of which are not found in the 1923 syllabus. Eleven percent of the propositions, then, on the papers from 1900 to 1924 are not in the 1923 syllabus.

In proving originals since 1923 the student would have to be familiar with two theorems that do not appear in the 1923 syllabus. Art.* 106 is used as the step in the proof of an original on the 1928 and 1924 papers. Art. 199 is used in the proof of an original on the 1930 and 1929 papers once and on the 1927 and 1924 papers twice. The student could do those originals if he had not known Articles 106 and 199, but he would have found them difficult.

Examining the papers from 1900 to 1924 it is found that Art. 73 was used in 1915; Art. 105 in 1922, in 1920, in 1911, and in 1907; Art. 106 used twice in 1921; Art. 199 in 1923, in 1922, in 1920, in 1915, in 1914, in 1913, in 1912, in 1911, in 1908, twice in 1902, and once in 1901; Art. 230 in 1916, in 1915, in 1914, in 1913, in 1906, in 1905, in 1902, and twice in 1901; Art. 233 in 1914, in 1913, in 1912; Art. 310 in 1906.

The 1923 syllabus does not entirely omit Articles 105 and 106 which deals with the relationship between the leg

* See Appendix C.

and hypotenuse in a 30°-60° triangle and the converse for it does say if the student is familiar with that relationship he will find some of the originals easier of solution, but it is not placed in the list of eighty-nine important propositions which are listed in the syllabus.

Art. 199 which states the relation between the central angle and the arc of the circle is used five times in the solution of originals between 1923 and 1931, but no mention is made of this proposition in the 1923 syllabus.

If these three propositions are omitted, it is found that in proving originals from 1900 to 1923 the student would use propositions as steps in proving originals 156 times. Fourteen or 9% of these times he would use as steps propositions which are not found in the 1923 syllabus.

Up to 1911 no constructions were called for. From 1911 to 1924 they appear quite frequently, and then they are omitted from 1924-1930.

During the first few years, namely 1901-1903, definitions were called for on the examinations. Once or twice since then the student has been asked to define "locus", but no other definitions have been asked. However, it seems reasonable to believe that a student would have difficulty in proving propositions or originals on any examination unless he knew fairly well the definitions of the terms he was using.

From 1923 to the present the paper contained but two book propositions. From 1911-1923 it varied from two to four, and from 1901-1911 there were from two to thirteen book propositions. However, in those years the student was given a choice so that never did he have to prove more than four or five propositions in one year.

The tables do not show the varying degrees of difficul-

ty of the originals during these years. This is extremely important, but it is impossible to show in the tables the degrees in difficulty of the originals. To the one solving the originals on the examination papers it was quite evident. The present originals, that is the originals given on the papers between 1924 and 1931, are comparatively simple to solve as compared with those given on the early examinations. The originals which appear on the examinations between 1901 and 1906 are very difficult. Those on the papers between 1906 and 1913 are almost as difficult in solution, but from 1913 to 1924 they become easier to solve. *

The present tendency is to devote more time to originals and less to book propositions than formerly. The most important tendency is the trend toward many originals which are comparatively simple to solve rather than few originals which are extremely difficult to solve.

What this means to the teacher and pupil is best summarized by quoting P. E. Elicker.** He says,"The content of the new requirements in geometry are essentially the same as before, but new emphasis has been placed on various parts of the new requirements. New starred list of propositions diminish demands on a pupil's memory and relieve the teacher of excessive drill. More time may be given to develop a real geometric understanding and an opportunity to do more of the computation of geometry".

* Note: The papers were analyzed from 1930 to 1901 rather than from 1901 to 1930, so it was not practice which made the originals on the later papers seem the easier. Practice gained in doing the papers from 1930 back should have made the originals on the earlier papers seem easier, but such was not the case.

* P. E. Elicker-Improvement in Teaching Algebra and Geometry Made Possible by the New College Board Requirements-Mathematics Teacher, Jan. 1925.

B. Tables from high school tests.

The replies to the letters requesting examinations from high schools were very limited. Of the sixty schools to which letters were sent only seventeen sent examinations in geometry.

Table

This table shows the type of the examination sent and the period covered by each of the nineteen schools.

	Final			End of Semester								
				Book one			Books 1, 2, 3			Book three		
School	NT*	E	NE	NT	E	NE	NT	E	NE	NT	E	NE
1	..	..	..	..	..	..	..	1	..	..	..	..
2	..	2	..	..	..	..	..	..	..	..	..	..
3	..	1	..	..	..	..	..	..	..	..	..	..
4	..	..	..	..	6	..	..	..	..	..	..	..
5	..	1	..	..	..	..	..	..	..	..	..	..
8	1	1	..	..	1	..	..	..	..	..	..	..
9	..	1	..	..	..	..	..	..	..	..	..	..
10	..	..	1	..	..	..	..	..	..	..	..	..
11	..	1	..	..	..	..	..	..	..	..	..	..
12	..	..	..	..	1	..	..	..	..	..	..	..
13	..	..	..	..	1	..	..	..	..	..	..	..
14	..	..	..	..	..	..	..	..	..	..	1	..
15	..	1	..	..	..	..	..	..	..	..	..	..
16	..	..	..	..	..	1	..	..	..	..	..	..
17	1	..	..	..	..	..	..	..	..	..	..	..
18	..	..	..	1	..	..	..	..	..	..	..	..
19	..	..	..	..	1	..	..	..	..	..	..	..
total	12	8	1	1	10	1		1			1	

*NT new-type examination.
E essay-type examination.
NE partly new-type and partly essay-type examination.

It was decided to analyze the final examinations received from these schools to determine if they were "in line with"-so to speak- as the present tendencies determined from the College Board examinations.

Table III

Proofs of these propositions were asked.

College Board No.*	School 2a	2b	3	7	8	9	10**	11	15	
11*	..	..	..	1	..	1	1	1	..	4
20*	..	..	..	..	..	1	..	..	..	1
23*	..	..	1	..	..	1	..	..	1	3
31*	..	..	..	..	..	1	..	..	..	1
41*	..	..	1	..	..	..	..	..	..	1
50	..	..	..	..	..	..	..	..	1	1
47*	..	..	1	..	..	..	..	..	..	1
51	1	..	..	1	..	..	..	..	..	1
58	..	..	1	..	..	..	..	..	..	1
59*	..	1	..	..	..	..	..	..	..	1
60*	1	..	..	1	1	..	..	..	..	3
61*	..	..	..	..	..	..	..	..	1	1
66*	..	..	..	..	1	..	..	..	..	1
72*	..	..	..	1	..	..	..	1	..	2
76*	..	..	..	..	..	..	..	1	..	1
77*	1	..	..	..	..	..	..	..	..	1
78*	1	..	..	..	..	..	..	..	..	1
81*	1	..	..	..	..	..	..	..	..	1
total	4	3	4	4	2	5	1	3	4	30

* Appendix B.

** The examination from school 10 was partly new-type and partly essay.

Table III

This table shows what theorems were used in proving originals given in the essay type finals received from the schools.

College Board No.*	School 2a	2b	3	7	8	9	11	15	Total
2*	..	..	..	..	..	1	..	..	1
3*	..	..	..	1	..	..	2	..	3
5*	..	..	..	..	1	..	..	..	1
10*	..	..	..	..	..	2	..	..	2
11*	..	..	..	..	..	1	..	..	1
19	..	..	..	..	..	2	..	..	2
24	1	..	..	1	..	..	1	..	3
25	1	1	..	1	..	..	.	..	3
41*	..	..	..	..	1	..	..	..	1
44	..	..	..	..	..	..	..	1	1
49	..	1	..	..	..	..	..	..	1
59*	..	1	..	..	..	..	..	1	2
66*	1	2	4	..	2	3	2	1	15
68	..	..	..	..	1	..	..	..	1
72*	1	..	..	..	..	2	1	..	4
73	..	..	..	..	..	..	1	1	2
77*	1	..	..	..	1	..	..	..	2
81	..	..	..	..	..	1	..	..	1
87	1	..	..	1	1	..	..	..	3
88	3	2	3	1	1	2	..	1	13
Art.105**	..	..	1	1	2	..	..	1	5
Art.106	1	..	1	..	..	1	2	..	5
Art.199	..	1	3	1	1	..	2	1	9
Art. 230	1	1	1	1	1	..	..	..	5
Art. 230B	..	..	..	..	1	..	..	..	1
total	11	9	13	8	13	15	11	7	87

The following constructions were used.

	2a	2b	3	7	8	9	11	15	Total
1	..	..	1	..	..	..	..	..	1
2	..	..	..	1	..	..	..	..	1
3	..	..	1	..	..	..	..	..	1
4	..	..	..	1	..	..	..	..	1
5	..	..	..	1	..	..	..	..	1
9	..	..	..	1	..	..	..	..	1
14	..	..	1	..	..	..	..	..	1
16	..	1	..	..	..	..	..	..	1
total		1	3	4					8

* Appendix B.
** Appendix C.

Table IV

This table shows what theorems were used in proving originals and answering new-type test questions on the two new-type finals and the partly new-type-partly essay-type final from the schools.

College Board No.	School 8	School 10	School 17	total
3*	..	1	1	2
10*	..	..	1	1
19	..	..	1	1
21*	..	..	2	2
23*	..	..	2	2
24	..	..	1	1
25	1	3	..	4
41*	..	1	..	1
49	1	..	..	1
52	..	1	..	1
57	..	1	..	1
58*	2	..	..	2
58*	1	..	..	1
60*	1	1	..	2
66*	1	..	..	1
68	1	..	..	1
69	1	3	..	4
70*	1	..	..	1
72*	2	2	1	5
73	1	..	..	1
76*	1	..	..	1
77*	2	..	..	2
81	1	..	..	1
82	2	..	..	2
86	..	1	..	1
87	1	2	..	3
88	3	1	..	4
Art.106**	2	1	..	3
Art.230	4	..	..	4
total	29	18	9	56

The following terms were called for as definitions.

Def.	School 10	School 17
Axiom	..	1
Complementary angle	3	2
Hypotenuse	1	..
Isosceles trapezoid	..	1
Median	..	2
Quadrilateral	1	2
Rectangle	1	3
Scalene triangle	..	1
Square	..	2
Supplementary angle	2	4

Constructions

Con. No.	School 8	School 10	School 17	
1	1	1	2	4
15	1	1	1	3
total	2	2	3	7

* Appendix B.

**Appendix C.

1. Summary.

The essay type of examination received from the high schools did not differ materially from the College Entrance Board examinations. Eighty-seven times were propositions used as steps in the proofs of originals and with the exception of Articles 105, 106, and 199* only eight times, or 9.2%, were propositions used which are not in the 1923 syllabus.

The new-type final examination did not call for any book proposition to be proved by the student.

Only twice was the proof of a book proposition demanded which was not a starred theorem in the 1923 syllabus.

The new-type final examinations contained many definitions and most of them were repeated more than once in the paper.

The teachers in those schools sending the essay-type final examination seem to feel the importance of Articles 105, 106, 199* quite as much as the College Entrance Examination Board, for on the eight examinations analyzed these three propositions were used nineteen times in the solution of originals.

The schools stress constructions much more than the College Entrance Examination Board.

The schools from which we received examinations seem to follow quite closely the requirements of the College Entrance Examination Board which indicates that these schools may stress college preparatory mathematics.

The present tendencies in plane geometry seem to follow the recommendations of the National Committee on Reorganization in Mathematics and the College Entrance Examination Board inasmuch as an analysis of examinations received indicate that the number of book propositions have been great-

* Appendix C.

ly reduced and the emphasis shifted to many originals of a simple type rather than to a few of a difficult type. Apparently it is recognized that many constructions belong in intuitive geometry for only a few appeared in the papers that were analyzed in plane demonstrative geometry.

V. CONSIDERATION OF THE PRESENT TENDENCIES IN ALGEBRA.

A. Summary of progress in last quarter century.

From the introduction of Algebra into the American Schools in the 18th century until the beginning of the 1900's the aims in teaching the subject were mainly cultural or disciplinary and the content more or less fixed by tradition. Simons(1) points out this fact by saying: "Nowhere are there formal indications that a practical need for Algebra actuated the teaching of it during this period, (the 18th century). The inclusion of this subject in the curriculum of a college of the 18th century, or the teaching of it as a special subject by some enthusiastic teacher, must be accounted for on the ground that it was done for the sake of the subject itself -- or for the theoretical aspects of fluxions".

As can be seen from the above, Algebra was at first purely a college subject. Gradually, however, it was introduced into the secondary schools but mainly for the same purpose. The cultural or the disciplinary aims were foremost. Up until about 1900 each college held its own entrance examinations and set its own standards and requirements but usually with but little regard for the secondary school and its needs. In 1900, however, the College Entrance Examination Board was organized for the purpose of unifying in some way the various entrance requirements. But even with this step in the direction

(1) "Introduction of Algebra into American Schools in the 18th Century" by Lao Genevra Simons. Bul. #18 U.S. Bur. of Ed. - 1924.

of unification there was little attention paid to the aims of teaching algebra and there was little change in the content --- the traditional topics still being required.

B. Recommendations of the National Committee on Mathematical Requirements. In 1916 the National Committee on Reorganization of Mathematics in Secondary Education was formed to consider what could and should be done to improve conditions. It was 1923 before the final report appeared but in it were many new ideas and good suggestions. Foremost of these was the change in the aims of teaching mathematics. The Committee recommended the teaching of mathematics first of all so that it would be of use to the student. Of course the former aims were not thrown to the winds but the idea of usefulness permeated the reorganization. Naturally this new note in the objectives of Algebra, (as one branch of mathematics), showed up in the revised content recommendations. The Committee stressed the fact that much of the material previously taught in Algebra was really of little or no use to the student and hence should be dropped --- and only those portions of real value should be retained. The Committee also recommended the addition of certain new topics and ideas such as Numerical Trigonometry, use of logarithms, the understanding of the graph, the formula, and the function concept.(1)

(1) Report of National Committee on Reorganization of Mathematics in Secondary Education. 1923.

About the same time the College Entrance Examination Board had a committee working along the same lines and in 1923 also issued a report and gave a list of recommendations. It is peculiar that these two reports coming out independently at about the same time should be so much alike. A study of the recommendations of each committee will show the great similarity in ideas and suggestions. In their report the College Board stated the revised requirements in the various fields of mathematics as applied to their entrance examinations. The College Board report as related to Elementary Algebra follows:-

C. Definition of the Requirements
in Elementary Algebra
as adopted by the College Entrance Examination Board.
1923. (1)

Elementary Algebra (Part I)--To Quadratics.

1. The meaning, use, evaluation, and necessary transformations of simple formulas involving ideas which the pupil is familiar with, and the derivation of such formulas from rules expressed in words.

The following are types of the formulas that may be considered:

$V = 4/3\pi r^3$ (the sphere). $A = \frac{1}{2}h(b+b')$ (the trapezoid).

$s = \frac{1}{2}gt^2$ (falling bodies) $A = p(1+rt)$ (simple interest).

$A = p(1+r)^t$ (compound interest).

(1) Document # 107 C.E.E.B. May 15, 1923.

2. The graph , and graphical representation in general. The construction and interpretation of graphs.

The following are types of the material adapted to this purpose; statistical data, formulas involving two variables, such as;

$$A = \pi r^2 \qquad \text{and} \qquad Y = x^2 + 3x - 2$$

formulas involving three variables, but considered for the case in which an arbitrary value is assigned to one of them as; $V = \pi r^2 h$ for a fixed value, say, of h.

3. Negative numbers; their meaning and use. This requirement includes the fundamental operations with negative numbers and the interpretation of a negative result in a problem, provided such an interpretation is germane to the problem.

4. Linear equations in one unknown quantity, and simultaneous linear equations involving two unknown quantities, with verification of results. Problems. The coefficients of a single linear equation in one unknown quantity may be literal fractions. In the case of simultaneous equations, literal coefficients are restricted to simple integral expressions, and to cases reducible to such expressions.

5. Ratio, as a case of simple fractions; proportion as a case of an equation between two ratios; variation; Problems.

6. The essentials of algebraic technique,

including; a. the four fundamental operations.

b. factoring of the following types:

1. Monomial factors.

2. the difference of two squares.

3. trinomials of the type x^2+px+q.

c. fractions, including complex fractions of the simple type. The requirement includes the following degree of difficulty in complex fractions;

$$\frac{p+\frac{a}{b}}{q+\frac{c}{d}} \qquad \frac{\frac{a+3b}{c-5d}}{\frac{a-3b}{c+5d}}$$

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{m}{n}-\frac{p}{q}}$$

d. numerical verification of the results secured under a. , b. , and c.

7. Exponents and radicals.

a. the proof of the laws for positive integral exponents.

b. the reduction of radicals, confined to transformations of the following types;

$$\sqrt{a^2b}=a\sqrt{b}, \quad \sqrt{\frac{a}{b}}=\frac{\sqrt{ab}}{b}, \quad \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$

$$\frac{1}{\sqrt[3]{a}}=\frac{\sqrt[3]{a^2}}{a}, \quad \frac{1}{\sqrt[3]{a^2}}=\frac{\sqrt[3]{a}}{a}$$

and to the evaluation of simple expressions involving the radical sign.

c. the meaning and the use of fractional exponents , limited to the treatment of the radicals that occur under b.

d. a process for finding the square root of a number, but no process for finding the square root of a polymomial.

8. Numerical Trigonometry. The use of the sine, cosine, and tangent in solving right triangles. The use of four place tables of natural trigonometric functions is assumed, but the teacher may find it useful to include some preliminary work with three place tables. It is important that the pupil should acquire facility in simple interpolation; in general emphasis should be laid on carrying the computation to the limit of accuracy permitted by the table.

Elementary Algebra (Part II.)---Quadratics and beyond.

1. Numerical and literal quadratic eqautions in one unknown quantity. Problems. The requirement includes the solution of the general quadratic equation

$$ax^2 + bx + c = 0$$

the conditions for the reality and for the distinctness of the roots, and the formulas for the sum and the product of the roots. Simple cases in which x is replaced by z^2 or by a linear binomial, and problems leading to quadratics, are also included. furthermore;

the interpretation of the graph of such an expression as;

$$x^2 - 3x + 5$$

meaning thereby the graph of the corresponding equation

$$y = x^2 - 3x + 5$$

2. The binomial theorem for positive integral exponents,with applications. It is not intended under this topic to include problems involving irrational numbers or surds or the expression of the expansion of the powers of a binomial having more than one fractional coefficient. Such simple applications as that to compund interest are included.

3. Arithmetic and geometric progression (series) The requirement is limited to the formulas for the nth term, the sum of the first n terms, the value of such an infinite decreasing geometric series as;

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\cdots.$$

and to, simple applications.

4. Simultaneous equations, consisting of one quadratic and one linear equation, or of two quadratic equations of the following types;

$$\begin{cases} ax^2 + by^2 = c \\ xy = h \end{cases} \qquad \begin{cases} a_1x^2 + b_1y^2 = c_1 \\ a_2x^2 + b_2y^2 = c_2 \end{cases}$$

These may be expressed in other forms, such as; $x^2 = (r+y)(r-y)$ $xy = r^2$

The coefficients may be integers, numerical fractions, or algebraic monomials. Graphical treatment is expected of equations of the types;

$$x^2 + y^2 = a^2 \quad ; \quad x^2 - y^2 = a^2 \quad ; \quad xy = a \quad ; \quad y^2 = ax \; .$$

6. Exponents and radicals.
   a. the theory and use of fractional, negative and zero exponents.
   b. the rationalization of the denominator in such expressions as

$$\frac{a + \sqrt{b}}{c - \sqrt{d}}$$

   c. the solution of such equations as

$$\sqrt{3 - 2x} - x = 30$$

7. Logarithms.
   a. the fundamental formulas.
   b. computation by four place tables.
   c. applications to the trigonometry of the right triangle.

VI. ANALYSIS OF COLLEGE BOARD EXAMINATIONS AND HIGH SCHOOL TESTS IN ELEMENTARY ALGEBRA.

A. Tables from College Board Examinations.

In order to determine whether the recommendations of the National Committee on Mathematical Requirements and of the College Board Committee are being followed in Secondary Mathematics at the present time --- or if not, what the present tendencies are --- the entire set of College Board Examinations from 1901 to 1930 have been analyzed.

The report of the National Committee and the 1923 syllabus of the College Board was used as a basis of the analysis.

The procedure used was as follows:-

First: the entire field of elementary algebra was divided up into units somewhat in the same manner as was done in the College Board Syllabus.

Second: these units were then sub-divided into the essential parts comprising the various types of operations, problems, and skills necessary.

Third: each year's examination was studied to note the types of questions and problems asked for. Whenever a question was noted a "-" mark was placed under the proper heading. For instance:- A question on factoring of a "perfect square" might be found on some examination. Then a "-" mark was put under "Perfect square" under the unit on Factoring for that year.

THE FORMULA.

Year.	Meaning.	Use.	Evaluation.	Transformation.	Derivation.	Problems.
1901						
1902						
1903						
1904						
1905						
1906		-	-	-		
1907		-	-	-		-
1908						
1909						
1910						
1911						
1912						
1913						
1914						
1915						
1916						
1917						
1918						
1919						
1920						
1921						
1922						
1923						
1924						
1925			-	-	-	
1926		-	-	-		
1927		-	-	-		
1928						
1929					-	-
1930	-				-	-

THE GRAPH.

Year.	Representing facts.	Representing dependence.	Solving problems.	Construction.	Solving equations.	Interpretation.
1901						
1902						
1903						
1904						
1905						
1906				-	-	
1907				-		
1908				-	-	
1909	-	-		-		
1910				-	-	
1911				-	-	-
1912				-	-	
1913						
1914				-		
1915						
1916				-	-	
1917						
1918				-	-	
1919				-	-	
1920				-	-	
1921						
1922						
1923				-	-	-
1924						
1925				-	-	
1926	-			-		
1927				-	-	
1928				-	-	
1929	-	-	-	-		-
1930					-	

LINEAR EQUATIONS.

Year.	One unknown. (Num.)	One unknown. (Lit.)	Fractional coefficients.	Simult. Lin. 2 unk. (Lit.)	Simult. Lin. 2 unk. (Num.)	Fractional.	Involving square root.	Problems.
1901	I				I			I
1902		I			I	I		I
1903	I	I			I			
1904	I			I		I		
1905					I			
1906				I	I			I
1907				I	I	I	I	I
1908					I	I		I
1909	I				I			I
1910					I			I
1911								I
1912	I			I		I		I
1913								
1914							I	
1915								
1916					I	I		I
1917	I				I	I	I	I
1918	I						I	I
1919					I	I		I
1920					I	I		I
1921								
1922								
1923								
1924					I			I
1925								I
1926	I							I
1927	I							I
1928	I					I		I
1929					I	I		I
1930	I				I	I		I

RATIO AND PROPORTION.

Year.	Simple fractions.	Equation between two ratios.	Variation.	Problems.	Third proportion, etc.	Mean proportion.	Alt. and Comp.	Permutation and Combination.
1901				I				I
1902					I	I	I	I
1903			I	I		I		I
1904								I
1905								
1906								
1907								
1908								
1909								
1910							I	
1911								
1912								
1913								
1914								
1915								
1916								
1917								
1918								
1919								
1920								
1921								
1922								
1923				I				
1924								
1925			I	I				
1926			I					
1927								
1928								
1929								
1930			I					

FUNDAMENTAL OPERATIONS.

Year.	Multiplication.	Division.	Addition.	Subtraction.	Parentheses.	Evaluation.	Long division.	Long multiplication.
1901							-	
1902						-	-	-
1903							-	
1904							-	-
1905								
1906								
1907								
1908								
1909								
1910								
1911								
1912								
1913								
1914								
1915								
1916						-		
1917		-			-	-		
1918								
1919								
1920								
1921								
1922								
1923								
1924								
1925								
1926	-				-			
1927						-		
1928								
1929						-		
1930	-		-	-				

FACTORING.

Year.	Common monomial.	Difference of two squares.	Trinomial of second degree.	Perfect square.	Polynomial.	Sum or diff. of two cubes.	Trinomial of third degree.	Trinomial of fourth degree.
1901					-			
1902		-	-		-	-		
1903	-	-	-		-	-		
1904								
1905			-					
1906								
1907			-				-	-
1908	-	-	-		-	-		
1909								
1910	-			-	-	-		
1911	-						-	-
1912								
1913								
1914			-		-			
1915								
1916			-		-			
1917			-	-	-			
1918		-	-		-	-		
1919	-		-		-	-		
1920		-	-	-	-			
1921								
1922								
1923			-		-	-		
1924	-	-	-					
1925		-	-					
1926	-	-		-				
1927	-	-	-					
1928		-						
1929	-	-	-					
1930			-	-	-			

FRACTIONS.

Year.	Multiplication.	Division.	Addition.	Subtraction.	Simplify.	Complex.(Simple.)	Evaluation.	L.C.M.	H.C.F.	Complex.(Fract.and Neg. exp.)
1901	I	I	I	I	I			I	I	I
1902	I	I	I	I	I			I	I	I
1903	I	I	I	I	I			I		I
1904								I	I	
1905								I	I	
1906					I	I				
1907								I	I	
1908			I					I	I	
1909								I		
1910										
1911					I					
1912					I	I				I
1913										
1914					I	I				
1915										
1916			I		I					
1917	I	I	I	I	I	I				
1918	I	I	I	I	I	I				
1919	I	I	I	I	I	I	I			
1920	I	I	I	I	I	I				
1921										
1922										
1923					I					
1924			I	I	I		I			
1925										
1926			I	I	I					
1927			I	I	I					
1928					I					
1929			I	I	I					
1930			I	I	I					

CHECKING.

Year.	Check.	Verify answer.	Show answer satisfies.
1901			
1902		1	
1903			
1904			
1905			
1906		1	
1907			
1908		1	
1909			
1910			
1911			
1912			1
1913			
1914			
1915			
1916			
1917			
1918	1	1	1
1919	1	1	1
1920	1	1	1
1921			
1922			
1923		1	
1924	1		
1925	1		
1926	1		
1927			
1928			
1929			
1930			

EXPONENTS AND RADICALS.

Year.	Law of exponents.	Reduction of radicals.	Fractional exponents.	Evaluation.	Square.	Cube.	Square root.(Number.)	Square root.(Polynomial.)
1901							-	-
1902	-	-	-	-			-	-
1903	-	-	-	-			-	-
1904	-	-	-					
1905	-	-	-					
1906								
1907		-	-	-				
1908		-	-					
1909		-	-					-
1910	-	-	-					
1911	-	-	-					
1912		-						
1913								
1914	-	-	-					
1915								
1916	-	-	-					
1917	-		-					
1918		-		-				
1919	-	-	-					
1920	-	-	-					
1921								
1922								
1923	-	-	-	-				
1924		-	-					
1925	-	-	-	-				
1926	-	-	-	-				
1927		-	-	-				
1928		-	-	-				
1929		-	-	-				
1930	-	-	-	-				

NUMERICAL TRIGONOMETRY.

Year.	Right triangle.	Four place tables.	Interpolation.	Computation.	Problems.
1901					
1902					
1903					
1904					
1905					
1906					
1907					
1908					
1909					
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1911					
1912					
1913					
1914					
1915					
1916					
1917					
1918					
1919					
1920					
1921					
1922					
1923					
1924	-			-	-
1925	-			-	-
1926	-			-	-
1927	-			-	-
1928	-			-	-
1929	-			-	-
1930	-			-	-

QUADRATIC EQUATIONS.

Year.	Numerical.(One unknown.)	Literal.(One unknown.)	Complete the square.	Factor.	Formula.	Problems.	Nature of roots.	Graphic solution.	Decimal coefficients.	Fractional and radical coef.
1901	-									-
1902	-					-	-			-
1903	-					-	-		-	-
1904	-					-	-		-	-
1905	-						-			-
1906				-		-		-		
1907			-			-		-		
1908	-					-				
1909										
1910		-								
1911	-						-			
1912	-					-	-			
1913										
1914							-			
1915										
1916										
1917	-					-	-		-	
1918	-			-			-			
1919	-						-			
1920	-									-
1921										
1922										
1923									-	
1924	-					-	-			
1925	-					-	-			
1926						-				
1927						-				
1928	-	-				-	-			
1929	-					-	-	-		
1930	-						-			

BINOMIAL THEOREM.

Year.	Expansion.	Specific term.	Formula.	Sum of specific terms.	Proof.	Evaluation.
1901	-				-	
1902	-	-		-		
1903	-	-				-
1904	-	-				
1905	-	-				
1906		-				
1907		-				
1908		-	-			
1909		-				
1910		-				
1911	-	-				
1912	-	-				
1913						
1914	-	-				
1915						
1916						
1917		-				
1918		-				
1919		-				
1920						
1921						
1922						
1923	-	-				
1924						
1925						
1926						
1927	-					
1928	-	-				
1929		-				
1930		-				

ARITHMETIC AND GEOMETRIC PROGRESSION.

Year.	Arithmetic progression.	Formula derivation.	Arithmetic mean.	Common difference.	Geometric progression.	Formula derivation.	Geometric mean.	Harmonic mean.	Sum of series.
1901			-		-	-			
1902	-	-	-		-	-		-	-
1903	-		-	-			-	-	-
1904			-				-	-	-
1905					-				-
1906					-				
1907	-								
1908		-			-				
1909			-		-	-			
1910	-	-							-
1911	-				-				
1912	-				-				
1913									
1914									
1915									
1916			-				-		
1917	-				-				
1918					-				
1919	-	-			-				-
1920	-								
1921									
1922									
1923					-	-			
1924					-	-			-
1925	-	-							-
1926									-
1927									-
1928	-								
1929	-					-			-
1930		-							

SIMULTANEOUS EQUATIONS.

Year.	Linear equations with three unknowns:-	a. With literal coef.	b. With numerical coef.	c. With algebraic poly.	One linear-one quadratic.	Problems.	Two quadratic.	Graph.
1901			-				-	
1902			-		-	-		
1903			-				-	
1904								
1905			-					
1906								
1907								
1908					-			-
1909					-			
1910					-			-
1911						-	-	
1912								
1913								
1914					-	-		
1915								
1916							-	-
1917								
1918			-		-			-
1919					-			-
1920					-			-
1921								
1922								
1923			-		-	-		
1924								
1925								-
1926			-		-			
1927					-			
1928			-		-			
1929					-			
1930			-				-	-

LOGARITHMS.

Year.	Fundamentals.	Computation.	Application to Trigonometry.
1901	I	I	
1902	I	I	
1903		I	
1904		I	
1905			
1906			
1907			
1908			
1909			
1910			
1911			
1912			
1913			
1914			
1915			
1916			
1917			
1918			
1919			
1920			
1921			
1922			
1923			
1924		I	
1925		I	I
1926		I	I
1927		I	
1928		I	I
1929		I	I
1930	I	I	I

NUMBER AND TYPE OF QUESTIONS.

Year.	Number given.	Number called for in three hours.	Choice or no choice.	Type of questions:-	a. Old type.	b. True-false.	c. Matching.	d. Analogies.	e. Completion.
1901	30	10	C		-				
1902	29	8	C		-				
1903	24	10	C		-				
1904	14	10	C		-				
1905	8	6	C		-				
1906	8	6	C		-				
1907	8	6	C		-				
1908	8	6	C		-				
1909	8	6	C		-				
1910	8	6	C		-				
1911	8	6	C		-				
1912	8	6	C		-				
1913									
1914	7	6	C		-				
1915									
1916	7	7	NC		-				
1917	8	8	NC		-				
1918	8	8	NC		-				
1919	8	8	NC		-				
1920	10	10	NC		-				
1921									
1922									
1923	10	10	NC		-				
1924	10	10	NC		-				
1925	10	10	NC		-	-			
1926	10	10	NC		-				
1927	10	10	NC		-				
1928	10	10	NC		-				
1929	26	26	NC		-				
1930	28	28	NC		-				

1. Summary.

The analysis of the College Board Examinations from 1901 to 1930 reveals some very interesting things about the progress and the tendencies in present day Algebra.

The first thing that is noticed is the general pruning which has taken place over the years-- a pruning which has reduced considerably the subject matter in elementary algebra--especially the complexity of the subject. Yet, at the same time new topics have been introduced.

The changes which have taken place have been in line with the newer ideas of the aims of teaching algebra. The old idea was that algebra should be taught for its cultural and disciplinary values. The modern age, however, brought about a change to the more practical values of algebra. Consequently, the effort has been to do away with the useless portions of the subject and teach only the portions really necessary for the pupil's future work. It was this idea which dominated the work of both the National Committee on Mathematical Requirements and the College Entrance Examination Board Committee on the new requirements in their reports of 1923

and is the one which dominates the present efforts at improvement in the teaching of algebra.

Probably the best way to show the trend throughout this period from 1901 to 1930 in the examinations of the College Board will be to take up each division of the subject separately, and discuss the changes which have taken place in that field.

<u>The formula</u>:- Although the stressing of the importance of the formula dates back to the report of 1923 it is interesting to find a very short period back in 1906 and 1907 when problems involving the formula appeared on the College Board Examinations. Moreover, the stress was laid on the use, evaluation, and transformation of the formula exactly as is recommended today. The only other times when the formula has appeared on the examinations are in 1925, 1926, 1927, 1929 and 1930. Hence the influence of the new requirements of the Board can be seen and also the evident trend to lay more stress on the importance of the formula and all it stands for and is capable of doing.

<u>The graph</u>:- The general impression is that the graph is more or less a new idea in elementary algebra--that is-- not very much attention was paid to it until the present age of statistical enlightenment.

However, the analysis of the examinations shows that throughout the entire period--at least from 1906 on to the present--questions involving the graph, its use, construction, meaning, and use in solving equations have appeared regularly. There is a tendency, however, toward the more practical aspects of the graph by stressing the importance of the interpretation of the graph and its use in representing facts and dependence.

Linear equations:- In the field of linear equations there seems to be little change over the entire period. Problems and equations to be solved have appeared on all the examinations in some form or other as would be expected when the importance of equations in algebra is considered. The change has been in the simplification of the requirements by doing away with the more complicated forms which after all were mainly puzzles rather than tests of the fundamentals. The tendency is toward a better understanding of the usefulness of the equation rather than merely manipulative skill.

Ratio and proportion:- With ratio and proportion little has been done since the days of 1901, 1902, 1903 and 1904 when questions were given on the phases such as mean proportional, third proportional, alternation and composition and such things as permutations and combinations. Since 1904 there have been few questions on

ratio and proportion except in 1910 and then in 1925, 1926, and 1930. The stress in the last three years mentioned was on variation and problems dealing with the relation of one quantity to another. This shows the trend toward the function concept in algebra which is receiving more and more attention as time goes on.

Fundamental operations:- The trend in this field has been away from the complicated operations such as long multiplication and long division to the more simple standard operations. The complicated problems involving parentheses have been dropped and more stress has been put on the evaluations of expressions.

Factoring:- It is in this field of factoring that a very great change has taken place. The old examinations stressed a very great number of cases of factoring as did the old texts. The old examinations during the period from 1901 to 1910 gave questions involving the factoring of complicated expressions such as long polynomials, trinomials of the third and fourth degree and more complicated forms of the sum and difference of two cubes, etc. The trend is very distinctly in accordance with the Board's recommendations in that the forms given for factoring are simpler and fall in the classes as given in the outline, namely; common monomial, difference of two squares, trinomial of the second degree and the perfect square.

<u>Fractions</u>:- The tendency in the field of fractions is distinctly for simplification---that is---doing away with the complicated examples involving L.C.M. and H.C.F. and very complex fractions with fractional and negative exponents. The examinations in late years have devoted their attention to the fundamental operations with fractions.

<u>Checking</u>:- Although there has been in the literature a great deal of talk about checking answers, there have been comparatively few signs of it on the examinations. That is---there have been few directions given to the pupils to check the answers that they obtained.

<u>Exponents and radicals</u>:- The trend here has been to get away from the more complicated forms of problems and questions---away from the square root of both the number and the polynomial. Although the Board has in its recommendations the square root of a number, this question has not come up on the Board examinations since 1904.

<u>Numerical Trigonometry</u>:- Here is an entirely new field in elementary algebra for prior to 1924 there were no questions on the subject on the Board examinations. Since 1924, however, there have been questions every year stressing the right triangle and computations and problems.

This shows the effect of the committee's recommendations in 1923.

Quadratic equations:- From complicated puzzle questions involving decimal, fractional, and radical coefficients the shift has been to mainly numerical equations in one unknown, problems, questions on the nature of the roots, and graphic solutions.

Binomial theorem:- The difficulty of the problems or examples involving the binomial theorem has decreased since the first few years of the Board's examinations, and the stress is now placed mainly on the expansion and determination of specific terms rather than on the more complicated aspects.

Arithmetic and geometric progression:- In fact almost the same thing might be said of the field of arithmetic and geometric progression---the effort to get away from the old type of trick problems---problems obviously for puzzle purposes, etc., is very noticeable. The trend is to the more practical aspects of the subject by stressing the fundamentals.

Simultaneous equations:- The graphic method of solution for these equations is noted more and more as the years go by, while the rest of the items seem to have remained practically the same.

Logarithms:- Here is noticed the same peculiar thing as with the formula--that is--that there was a period during the years 1901, 1902, 1903 and 1904 when there appeared problems and questions involving logarithms, but from then until 1924 there are no signs of the subject as shown by the utter lack of problems or questions calling for logarithms. Since the report of the National Committee on Mathematical Requirements and the College Board Committee report, however, there have been logarithm problems each year. Here again can be seen the great influence of these two bodies in determining the content in Secondary Mathematics.

Number of questions:- It is very interesting to note the trend in the number and kind, and quality, of the questions asked on the examinations throughout this period from 1901 to 1930. The trend, as may readily be seen from the chart regarding this point, is toward more questions, with no choice, in an attempt to adequately cover the field of elementary algebra and thereby really try to test the abilities of the candidate rather than just sample here and there as was done when only six questions were required in three hours. Now, as may be seen, the number of questions required in three hours is 28 and there is no choice.

It is interesting to note also that only once has the College Board experimented with the so-called "new type" examination----in 1925 they gave one question which was made up of a group of "true-false" questions.

B. <u>Tables from High School Tests</u>.

The same general purpose has guided the study of the tests given in High Schools as guided the study of the College Board Examinations.

The same forms were used in the analysis except that the number of the school sending the test took the place of the year on the College Board charts.

In the letter which was sent out to the various High Schools asking for the sample tests, the statement was made that the name of the school would not appear in any way in the thesis, so that it was necessary to resort to a number system to identify one school from another. The same number system was used throughout the study for the algebra and geometry examinations.

All the tests sent in were not of the same type. Hence a table was prepared to show the type of test sent in by each of the schools and the type of questions asked. Also on this chart was noted the matter of checking requirements.

THE FORMULA.

School.	Meaning.	Use.	Evaluation.	Transformation.	Derivation.	Problems.
1				-		
2		-	-	-	-	
3	-			-		
4			-	-	-	
5		-		-	-	
6			-	-		
7		-	-	-		
8			-	-		
* 9	-					
10				-		
11		-				-
12			-	-		
13						
14						
15						
16			-	-		

THE GRAPH.

School.	Representing facts.	Representing dependence.	Solving problems.	Construction.	Solving equations.	Intrepretation.
1		-		-	-	
2				-	-	-
3						
4						
5				-		
6					-	
7					-	-
8				-	-	-
* 9	-		-	-		
10				-	-	
11						
12	-			-		-
13						
14						
15						
16						

LINEAR EQUATIONS.

School.	One unknown.(Numerical).	One unknown.(Literal).	Fractional coefficients.	Simultaneous linear,2 unk.lit.	Simultaneous linear,2 unk.num.	Fractional.	Involving square root.	Problems.
1	-					-		-
2	-				-	-		-
3	-					-		-
4	-				-	-		-
5	-				-			-
6	-				-	-		-
7	-				-	-		-
* 8	-				-	-		-
9	-		-		-	-		-
10	-				-	-		-
11	-				-	-	-	-
12	-		-					
13								-
14					-	-		-
15	-				-	-		-
16	-							-

RATIO AND PROPORTION.

School.	Simple fractions.	Equation between two ratios.	Variation.	Problems.	Third proportional, etc.	Mean proportional.	Alt. and comp.	Permutations and combinations.
1								
2			-	-				
3			-					
4								
5								
6								
7			-	-				
8		-		-				
9								
10				-				
11								
12								
13								
14								
15								
16								

FUNDAMENTAL OPERATIONS.

School.	Multiplication.	Division.	Addition.	Subtraction.	Parentheses.	Evaluation.	Long division.	Long multiplication.
1	-							
2	-	-	-	-	-	-		
3	-	-	-	-	-	-		
4	-	-	-	-	-	-		
5	-	-	-	-	-			
6	-	-			-			
7								
8	-	-	-	-	-	-	-	-
* 9	-	-	-	-	-	-		
10							-	
11	-	-					-	
12			-	-	-	-	-	-
13	-	-	-			-	-	-
14								
15								
16	-	-	-	-	-	-	-	-

FACTORING.

School.	Common monomial.	Difference of two squares.	Trinomial of second degree.	Perfect square.	Polynomial.	Sum or difference of two cubes.	Trinomial of third degree.	Trinomial of fourth degree.
1	-	-	-	-	-			
2	-	-	-	-	-			
3	-	-	-	-				
4	-	-	-	-	-			
5	-	-	-	-				
6	-	-	-	-				
7		-	-		-			
8	-	-	-	-	-			
* 9								
10	-	-	-					
11			-					
12								
13	-		-		-			
14								
15								
16	-	-	-	-	-			

FRACTIONS.

School.	Multiplication.	Division.	Addition.	Subtraction.	Simplify.	Complex.(simple).	Evaluation.	L.C.M.	H.C.F.	Complex.(fractional and negative exponents.)
1	-	-	-	-	-	-				
2	-	-	-	-	-	-	-			
3										
4			-		-					
5										
6										
7					-					
* 8	-	-	-	-	-	-				
9										
10				-	-					
11			-	-	-					-
12										
13						-				
14										
15					-					
16		-			-					

EXPONENTS AND RADICALS.

School.	Law of exponents.	Reduction of radicals.	Fractional exponents.	Evaluation.	Square of polynomial.	Cube of polynomial.	Square root.(Number).	Square root(Polynomial).
1		-	-		-	-	-	-
2	-	-	-				-	-
3		-			-		-	
4		-						
5								
6	-	-						
7	-	-	-					
* 8	-	-	-					
9	-							
10		-					-	
11	-	-	-					
12								
13								
14	-	-	-					
15		-		-				
16	-	-	-	-				

NUMERICAL TRIGONOMETRY.

School.	Right triangle.	Four place tables.	Interpolation.	Computation.	Problems.
1					
2	-		-	-	-
3					
4					
5					
6	-			-	-
7			-	-	-
8	-		-	-	-
* 9					
10	-				-
11	-				-
12					
13					
14					
15					
16					

QUADRATIC EQUATIONS.

School.	Numerical - one unknown.	Literal - one unknown.	Complete the square.	Factor.	Formula.	Problems.	Nature of roots.	Graphic solution.	Decimal coefficients.	Fractional or radical coefficients.
1	-	-	-	-	-	-				
2	-				-	-				
3	-									
4	-									
5										
6	-									
7	-									
8										
*9										
10	-		-	-	-	-				
11	-					-				
12										
13										
14										
15	-		-		-					
16	-					-				

BINOMIAL THEOREM.

School.	Expansion.	Specific term.	Formula.	Sum of specific terms.	Proofs.	Evaluation.
1	-	-				
2	-	-				
3						
4						
5						
6						
7	-	-	-			
8	-					
* 9						
10						
11		-				
12						
13						
14						
15		-				
16						

ARITHMETIC AND GEOMETRIC PROGRESSION.

School.	Arithmetic progression.	Formula derivation.	Arithmetic mean.	Common difference.	Geometric progression.	Formula derivation.	Geometric mean.	Harmonic mean.	Sum of series.
1					-				
2					-				
3									
4									
5									
6									
7	-				-				
8					-				-
9									
10									
11	-				-				-
12									
13									
14									
15									
16									

SIMULTANEOUS EQUATIONS.

School.	In three unknowns:- a. Literal coefficients.	b. Numerical coefficients.	c. Algebraic monomials.	In two unknowns:- One linear- one quadratic.	Problems.	Two quadratic.	Graph.
1							
2		-		-			
3							
4							
5							
6							
7				-			
8							
* 9							
10							
11		-		-			
12							
13							
14					-		-
15				-	-		
16		-					

LOGARITHMS.

School.	Fundamental formulas.	Computation.	Application to Trig.
1			
2	-	-	
3			
4			
5			
6			
7	-	-	
8			
* 9			
10			
11			
12			
13			
14			
15		-	
16			

TYPES OF TESTS -- CHECKING.

School.	Types of tests.	Types of questions.	Checking.
1	Topical.	Old type.	-
2	Finals-Midyear.	Old type-matching.	-
3	Finals-Makeup.	Old type-T.F.	
4	Quarterly.	Old type.	
5	Final (?).	Old type.	
6	Final.	Old type-New type(?)	-
7	Finals-Midyear.	Old type.	-
8	Finals-Midyear.	Old type-T.F.	-
9	Finals.	Old type-T.F.-Compl.	
10	Finals-Midyear.	Old type-Ind.Diff.	
11	Finals.	Old type.	
12	Midyear.	Old type.	-
13	Midyear.	Old type.	
14	Topical.	Old type.	
15	Final-Quarterly.	Old type-Diagnostic.	
16	Midyear.	Old type.	-

1. Summary.

The analysis of the tests given in high schools, even if there are but a few tests available as the result of our requests to the sixty schools, shows some interesting things. Also--a comparison of these high school tests with the College Board Examinations and with the recommendations of the Board, shows up certain similarities, and also variances.

Probably the best way to consider these examinations will be as the College Board examinations were considered-that is-each topic separately.

The formula:- The formula seems to be stressed considerably in the high schools--and rightly so--for it is one of the most important phases of elementary algebra and gives the student a good grasp upon the function concept--and brings before the student material and methods applicable to every day life. The emphasis is laid mostly on the evaluation and transformation of the formula with but little problem work in evidence.

The graph:- The graph is given a considerable amount of prominence in the high school tests analyzed--although there were nearly half of the high schools without any questions involving graphs. The high schools that did mention graphs made quite a feature of them, which shows how teachers vary in their ideas on what topics should

be emphasized and what ones given a minor place. The stress is laid mainly on the construction and interpretation of graphs from a statistical point of view---and the use of graphs in the solving of equations.

Linear equations:- The field of linear equations is one field of elementary algebra where the tests given in high schools and the College Board Examinations line up very closely. In practically every school great emphasis is put upon linear equations---mainly those in one unknown (numerical) and simultaneous linear equations (numerical) with variety in form by including fractional types. Problems involving the use of linear equations are always numerous. Here, as in the later College Board Examinations, the recommendations are followed by omitting the more complicated forms such as those involving square root and fractional coefficients.

Ratio and proportion:- The field of ratio and proportion is rather neglected in high school algebra; especially proportion from the standpoint of an equality between ratios; although, of course, this might be hard to recognize as such on a final examination paper. However, the high school tests and the later College Board Examinations correlate very well in their emphasis in this field.

Fundamental operations:- The subject of fundamental operations is covered more thoroughly on the high school tests than on the College Board Examinations.

However, this is but natural when one considers the type and purpose of each examination. In the high school the purpose is to teach the fundamentals--the College Board Examinations try to determine how well the student understands and can use his knowledge of algebra.

Factoring:- The types of factoring found on the high school tests are limited to common monomial, difference of two squares, trinomial of second degree, and the polynomial, and perfect square. This is in accordance with the trend as shown by the College Board Examinations.

Fractions:- With fractions some schools did considerable while others did practically nothing on the tests. The evident trend is, however, to limit the work with fractions to the simpler operations including addition, subtraction, multiplication and division, combined with work on simplification of fractions and work with the simpler types of complex fractions. There were no questions dealing with L.C.M. and H.C.F. and only one on a complex fraction with fractional and negative exponents. This is in accordance with the recommendations of the National Committee and the College Board.

Exponents and radicals:- Contrary to the recommendations some of the schools which sent tests are doing work with the square and the cube of polynomials and the square root and the cube root of polynomials. The other schools are limiting their work to the simpler

reductions of radicals, the law of exponents, the meaning of fractional exponents, and some evaluation work.

<u>Numerical trigonometry</u>:- Six of the sixteen schools sending tests did some work with numerical trigonometry. This is not a very high percentage, 37½%, and indicates that there are many teachers who evidently do not believe in placing work in trigonometry in a course in algebra. Since this phase of algebra is the one which has the greatest amount of interest to the average student because of its direct application to situations known to the student, it seems that special pains should be taken to introduce this work into the algebra course if for nothing else but to create interest.

<u>Quadratic equations</u>:- The work with quadratic equations in the high school tests was limited mostly to simple quadratics in one unknown (numerical) with some problem work, and a few questions which specified the method to be used in the solution of the given equations. One phase of the work which received little or no attention was work with literal equations and graphic solutions. In all the work on equations it is noted that there is a distinct aversion to literal equations of all types.

<u>The binomial theorem</u>:- The work in the field of the binomial theorem is limited mainly to the expansion of the given expression and the finding of some specific term of the expansion without fully expanding.

Arithmetic and geometric progression:- The work on progressions seems to be quite limited although this may be accounted for since some of the tests received were for first year algebra and some were for review algebra. The tests from a first year algebra class would not be likely to have any work on progressions on it, while the tests from the more advanced classes would probably have such questions. At any rate, the work when it appeared was mainly on straight series expansions and on the sum of a series.

Simultaneous equations:- In the unit on simultaneous equations the work was limited to equations with numerical coefficients--equations in two unknowns, one of which was a linear equation and the other a quadratic--and equations in three unknowns. There were some problems and one question asking for a graphic solution.

Logarithms:- Little appeared on the high school tests as far as logarithms are concerned. A few questions on the fundamental formulas and some computations are about all that are given on the tests studied.

Checking:- It seems peculiar that so little stress is laid on the important function of checking in the tests studied. The same thing was true of the College Board Examinations. Much talk has been heard about the necessity for checking algebraic work--and--so it seems that some results should be seen in the tests given in the

line of requests for checking. Only 44% of the tests made any reference to checking of answers.

Types of tests:- As can be seen from the table headed "Types of Tests", the main stress is still on the old type tests. There is evidence here and there of the new type tests but the applications of the newer types have been more or less half-hearted. That is--there have been few tests sent to us which were mainly the new type. The new type questions appeared as a part of an old type test.

There was little evidence of an allowance for individual differences on these tests. Only one test showed any arrangement calculated to allow for these differences.

VII. GENERAL CONCLUSIONS AND SUGGESTIONS FOR IMPROVEMENT.

The purpose of this thesis was to determine the past and present tendencies in secondary school mathematics and to see if the present tendencies in classroom practices as determined by a study of College Entrance Board Examinations and tests given in High School were "on the same track", so to speak, as the trends.

The results of the study in the field of Plane Geometry and Elementary Algebra give rise to the following conclusions:-

The trend in geometry is to make a course of study which the student of average ability may pursue profitably. The content of the subject has changed little since 1900, but the emphasis has shifted from book propositions to originals. In order to provide for the average student the tendency has been to introduce many originals of a simple nature rather than few but difficult ones. Many obvious propositions have been accepted without proof or as postulates. The theory of limits has been eliminated and many more applied problems substituted in its place.

There is a tendency to introduce originals of a practical nature, such as those which deal with the relationships existing in right triangles and the ratio between similar figures. There is a tendency to do away with many construction problems which is no doubt due to the fact that many constructions have been placed in intuitive geometry

where they belong.

A study of the tests given in high school geometry indicates that the present tendency is to follow quite closely the recommendations of the National Committee on Mathematical Requirements and of the College Entrance Examination Board, although there is a slight tendency to stress construction problems more than the National Committee recommends.

The trend in elementary algebra is to a simpler course of study conceived primarily for the student of average ability who is not necessarily going to college but who desires to, and should, obtain benefit from the study of the subject. However, this trend toward simplification does not necessarily mean a lowering in the amount of subject matter. The shrinkage resulting from cutting out the more difficult portions of the different topics is overcome by the addition of other topics.

Hence, the trend has been to do away with such things as complicated cases of factoring, cases of square and cube root of polynomials, complex fractions, much of the work in the binomial theorem, progressions, and the like, which after all were of little use to the average student, and to substitute for the deleted material such subject matter as graphs, numerical trigonometry, the formula, and logarithms.

There has also been a decided shift in the

emphasis in the presentation of the subject. Previously the subject was taught because it was felt that the mind training obtained was valuable and that there was a transfer of this training to other fields of activity. Now, however, the subject is taught because it is felt that the knowledge gained will be of practical value to the student.

This new emphasis is responsible for the stress now being laid on the portions of the subject which will be useful such as numerical trigonometry, the formula, and the function concept.

The study of the College Entrance Board Examinations reveals all this as the examinations are taken over the years from 1901 to 1930. The recommendations of the various committees have been rather carefully followed during this transition period.

The study of the tests given in high schools has shown, however, that although the trend is going in a certain direction that some of the high schools are "on the track" all right----but that some of them seem to be "running with only one wheel on the track", so to speak. By that is meant that although some schools are definitely in step with the newer ideas in secondary mathematics teaching, that other schools are still teaching obsolete material, teaching that material in the old manner with but little regard for the interests of the pupil, or his needs.

This is shown by the fact that some schools still include in their tests such material as square root and cube root of polynomials, very complex fractions with fractional and negative exponents, long division of polynomials and long multiplication of polynomials.

Neither the College Entrance Board Examinations nor the high school tests show any marked tendency toward the use of the new type or the standardized tests.

Therefore--a suggested improvement in the teaching of secondary mathematics in the High School would be to take account of stock to see whether the newer ideas in teaching of the subject were being carried out, whether obsolete material were being presented, and whether it was being presented in the old cut-and-dried manner.

Individual differences should be given more consideration through a more wide-spread use of new type tests.

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Appendix A

List of Propositions in Plane Geometry Recommended by The National Committee on Mathematical Requirements. *

I. Assumptions and theorems for informal treatment.

This list contains propositions which may be assumed without proof(postulates) and theorems which it is permissible to treat informally.

1. Through two distinct points it is possible to draw one straight line, and only one.
2. A line segment may be produced to any desired length.
3. The shortest path between two points is the line segment joining them.
4. One and only one perpendicular can be drawn through a given point to a given straight line.
5. The shortest distance from a point to a line is the perpendicular distance from the point to the line.
6. From a given center and with a given radius one and only one circle can be described in a plane.
7. A straight line intersects a circle in at most two points.
8. Any figure may be moved from one place to another without changing its shape or size.
9. All right angles are equal.
10. If the sum of two adjacent angles equals a straight angle, their exterior sides form a straight line.
11. Equal angles have equal complements and equal supplements.
12. Vertical angles are equal.
13. Two lines perpendicular to the same line are parallel.
14. Through a given point not on a given straight line, one straight line, and only one, can be drawn parallel to the

* The Reorganization of Mathematics in Secondary Education pp. 79-87.

given line.

15. Two lines parallel to the same line are parallel to each other.

16. The area of a rectangle is equal to the base times the altitude.

II. Fundamental theorems and constructions.

It is recommended that theorems and constructions (other than originals) to be proved on college entrance examinations be chosen from the following list. Originals and other exercises should be capable of direct solution by reference to one or more of these propositions and constructions.

A. Theorems

1. Two triangles are congruent if (a) two sides and the included angle of one are equal, respectively, to two sides and the included angle of the other; (b) two angles and a side of one are equal, respectively, to two angles and the corresponding side of the other; (c) the three sides of one, are equal respectively, to the three sides of the other.

2. Two right triangles are congruent if the hypotenuse and one other side of one are equal, respectively, to the hypotenuse and another side of the other.

3. If two sides of a triangle are equal, the angles opposite these sides are equal; and conversely.

4. The locus of a point (in a plane) equidistant from two given points is the perpendicular bisector of the line joining them.

5. The locus of a point equidistant from two given intersecting lines is the pair of lines bisecting the angles formed by these lines.

6. When a transversal cuts two parallel lines, the alternate interior angles are equal; and conversely.

7. The sum of the angles of a triangle is two right angles.

8. A parallelogram is divided into congruent triangles by either diagonal.

9. Any (convex) quadrilateral is a parallelogram (a) if the opposite sides are equal; (b) two sides are equal and parallel.

10. If a series of parallel lines cut off equal segments on one transversal they cut off equal segments on any transversal.

11.(a) The area of a parallelogram is equal to the base times the altitude.

(b) The area of a triangle is equal to one half the base times the altitude.

(c) The area of a trapezoid is equal to half the sum of its bases times its altitude.

(d) The area of a regular polygon is equal to half the product of its apothem and perimeter.

12.(a) If a straight line is drawn through two sides of a triangle parallel to the third side it divides these sides proportionally.

(b) If a line divides two sides of a triangle proportionally it is parallel to the third side. (Proofs for commensurable cases only.)

(c) The segments cut off on two transversals by a series of parallels are proportional.

13. Two triangles are similar if (a) they have two angles of one equal, respectively, to two angles of the other; (b) they have an angle of one equal to an angle of the other and the including sides are proportional; (c) their sides are respectively proportional.

14. If two chords intersect in a circle, the product of the segments of one is equal to the product of the segments of the other.

15. The perimeters of two similar polygons have the same

ratio as any two corresponding sides.

16. Polygons are similar, if they can be decomposed into triangles which are similar and similarly placed; and conversely.

17. The bisector of an (interior or exterior) angle of a triangle divides the opposite side (produced if necessary) into segments proportional to the adjacent sides.

18. The areas of two similar triangles (or polygons) are to each other as the squares of any two corresponding sides.

19. In any right triangle the perpendicular from the vertex of the right angle on the hypotenuse divides the triangle into two triangles each similar to the given triangle.

20. In a right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

21. In the same circle or equal circles, if two arcs are equal, their central angles are equal; and conversely.

22. In any circle angles at the center are proportional to their intercepted arcs. (Proof for commensurable case only.)

23. In the same circle or in equal circles, if two chords are equal their corresponding arcs are equal; and conversely.

24.(a) A diameter perpendicular to a chord bisects the chord and the arcs of the chord. (b) A diameter which bisects a chord (that is not a diameter) is perpendicular to it.

25. The tangent to a circle at a given point is perpendicular to the radius at that point; and conversely.

26. In the same circle or in equal circles, equal chords are equally distant from the center; and conversely.

27. An angle inscribed in a circle is equal to half the central angle having the same arc.

28. Angles inscribed in the same segment are equal.

29. If a circle is divided into equal arcs, the chords of these arcs form a regular inscribed polygon and tangents at

the points of division form a regular circumscribed polygon.

30. The circumference of a circle is equal to 2 pi r. (Informal proof only.)

31. The area of a circle is equal to pi r^2. (Informal proof only.)

B. Constructions

1. Bisect a line segment and draw the perpendicular bisector.
2. Bisect an angle.
3. Construct a perpendicular to a given line through a given point.
4. Construct an angle equal to a given angle.
5. Through a given point draw a straight line parallel to a given straight line.
6. Construct a triangle, given (a) the three sides; (b) two sides and the included angle; (c) two angles and the included side.
7. Divide a line segment into parts proportional to given segments.
8. Given an arc of a circle, find its center.
9. Circumscribe a circle about a triangle.
10. Inscribe a circle in a triangle.
11. Construct a tangent to a circle through a given point.
12. Construct the fourth proportional to three given line segments.
13. Construct the mean proportional between two given line segments.
14. Construct a triangle (polygon) similar to a given triangle (polygon).
15. Construct a triangle equal to a given polygon.
16. Inscribe a square in a circle.
17. Inscribe a regular hexagon in a circle.

III. Subsidiary list of propositions.

1. When two lines are cut by a transversal, if the corresponding angles are equal or if the interior angles on the same side of the transversal are supplementary, the lines are parallel.

2. When a transversal cuts two parallel lines, the corresponding angles are equal, and the interior angles on the same side of the transversal are supplementary.

3. A line perpendicular to one of two parallels is perpendicular to the other also.

4. If two angles have their sides respectively parallel or respectively perpendicular to each other, they are either equal or supplementary.

5. Any exterior angle of a triangle is equal to the sum of the two opposite interior angles.

6. The sum of the angles of a convex polygon of n sides is 2(n - 2) right angles.

7. In any parallelogram (a) the opposite sides are equal; (b) the opposite angles are equal; (c) the diagonals bisect each other.

8. Any (convex) quadrilateral is a parallelogram, if (a) the opposite angles are equal; (b) the diagonals bisect each other.

9. The medians of a triangle intersect in a point which is two thirds of the distance from a vertex to the mid-point of the opposite side.

10. The altitudes of a triangle meet in a point.

11. The perpendicular bisectors of the sides of a triangle meet in a point.

12. The bisectors of the angles of a triangle meet in a point.

13. The tangents to a circle from an external point are

equal.

14. (a) If two sides of a triangle are unequal, the greater side has the greater angle opposite it, and conversely.

(b) If two sides of a triangle are equal respectively to two sides of another triangle, but the included angle of the first is greater than the included angle of the second, then the third side of the first is greater than the third side of the second, and conversely.

(c) If two chords are unequal, the greater is at the less distance from the center, and conversely.

(d) The greater of two minor arcs has the greater chord, and conversely.

15. An angle inscribed in a semicircle is a right angle.

16. Parallel lines cutting a circle intercept equal arcs on the circle.

17. An angle formed by a tangent of a circle and a chord drawn through the point of contact is measured by half the intercepted arc.

18. An angle formed by two intersecting chords is measured by half the sum of the intercepted arcs.

19. An angle formed by two secants or by two tangents to a circle is measured by half the difference between the intercepted arcs.

20. If from a point without a circle a secant and a tangent are drawn, the tangent is the mean proportional between the whole secant and the external segment.

21. Parallelograms or triangles of equal bases and equal altitudes are equal.

22. The perimeters of two regular polygons of the same number of sides are to each other as their radii and also as their apothems.

Appendix B

List of Propostions in Plane Geometry Adopted by The College Entrance Examination Board. *

Standard theorems which the candidate should be able to demonstrate on the examination are starred.

In the case of unstarred propositions, the candidate will be expected to be familiar with their content, so as to be able to answer questions about their substance or use them as a basis for solving originals, but he will not be assumed to have their demonstrations in mind.

1*. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

2*. Two triangles are congruent if a side and the adjacent angles of one are equal respectively to a side and the adjacent angles of the other.

3*. If two sides of a triangle are equal, the angles opposite these sides are equal.

4*. If two angles of a triangle are equal, the sides opposite these angles are equal.

5*. Two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.

6*. Not more than one perpendicular can be drawn to a given line from a given external point.

7. Two lines, in the same plane perpendicular to the same line, are parallel.

If a line is perpendicular to one of two parallel lines it is perpendicular to the other also.

9. Two right triangles are congruent if the hypotenuse and a side of one are equal respectively to the hypotenuse and a side of the other.

* College Entrance Board Syllabus in Plane Geometry pp. 9-16. (Document No. 108, May 15, 1923.)

10*. Two right triangles are congruent if the hypotenuse and an adjacent angle of one are equal respectively to the hypotenuse and adjacent angle of the other.

11*. If two parallel lines are cut by a transversal, the alternate interior angles are equal.

12. If two parallel lines are cut by a transversal, the corresponding angles are equal.

13. If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

14. If two lines are cut by a transversal, so that a pair of alternate angles are equal, the lines are parallel.

15. If two straight lines are cut by a transversal so that a pair of corresponding angles are equal, the lines are parallel.

16. If two straight lines are cut by a transversal so thattwo interior angles on the same side of the transversal are supplementary, the lines are parallel.

17. If two angles have their sides respectively parallel, or respectively perpendicular, they are either equal or supplementary.

18. The opposite sides of a parallelogram are equal and the opposite angles are equal.

19. The diagonals of a parallelogram bisect each other.

20*. If the opposite sides of a figure are equal, the figure is a parallelogram.

21*. If two sides of a quadrilateral are equal and parallel the figure is a parallelogram.

22*. If three or more parallels intercept equal segments on one transversal, they intercept equal segments on every transversal.

23* The sum of the angles of a triangle is equal to two

right angles.

24. An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

25. The sum of the angles of a convex polygon of n sides is 2(n - 2) right angles.

26. If two sides of a triaggle are unequal, the angles opposite these sides are unequal, and the angle opposite the greater side is the greater.

27. If two angles of a triangle are unequql, the sides opposite these angles are unequal, and the side opposite the greater angle is the greater.

28. If two sides of one triangle are equal respectively to two sides of another triangle, but the included angle of the first is greater than the included angle of the second, then the third side of the first is greater than the third side of the second.

29. If two sides of one triangle are equal respectively to two sides of another triangle, but the third side of the first is greater than the third side of the second, then the angle opposite the third side of the first is greater than the angle opposite the third side of the second.

30*. The locus of points equidistant from two given points is the perpendicular bisector of the line joining them.

31*. The locus of points equidistant from two given intersecting lines is the pair of lines bisecting the angles formed by the given lines.

32. The perpendicular bisectors of the sides of a triangle meet in a point.

33. The bisectors of the angles of a triangle meet in a point.

34. The altitudes of a triangle meet in a point.

35. The medians of a triangle meet in a point, which is

two-thirds of the distance from any vertex to the mid-point of the opposite side.

36*. Through any ghree given points not lying in a straight line one circle, and only one, can be drawn.

37. In the same circle, or in equal circles, equal central angles intercept equal arcs, and equal arcs determine equal central angles.

38. In the same circle, or in equal circles, two central angles are proportional to their intercepted arcs.

39. In the same circle, or in equal circles, if two arcs are equal, their chords are equal; and, conversely, equal chords determine equal minor arcs and equal major arcs.

40*. In the same circle, or in equal circles, equal chords are equidistant from the center, and chords equidistant from the center are equal.

41*. A diameter perpendicular to a chord of a circle bisects the chord and the arcs determined by the chord.

42. A diameter which bisects a chord (that is not a diameter) is perpendicular to it.

43. A line perpendicular to a radius at its outer extremity is tangent to the circle.

44. The tangent to a circle at a given point is perpendicular to the radius drawn to that point.

45. If tangents to a circle from an external point are drawn they make equal angles with the line joining the given point to the center, and their segments from the given point to the points of contact are equal.

46. Two parallel secants, or two parallel tangents, or a tangent and a secant parallel to it, intercept equal arcs on a circle.

47*. An inscribed angle is measured by half the intercepted arc.

48. Angles inscribed in the same segment, or in equal segments are, equal.

49. An angle inscribed in a ~~right~~ semicircle is a right angle.

50. An angle formed by a tangent and a chord of a circle is measured by half the intercepted arc.

51. An angle formed by two intersecting chords of a circle is measured by half the sum of the intercepted arcs.

52. An angle formed by two secants of a circle, or by two tangents, or by a secant and a tangent, intersecting at a point outside the circle, is measured by half the difference between the intercepted arcs.

53. If in the same circle, or in equal circles, two minor arcs are unequal, the greater arc determines the greater chord.

54. If in the same circle, or in equal circles, two chords are unequal, the greater chord determines the greater minor arc.

55. If in the same circle, or in equal circles, two chords are unequal, the shorter is at the greater distance from the center.

56. If in the same circle, or in equal circles, two chords are unequally distant from the center, the one at the greater distance from the center is the shorter.

57. A line parallel to one side of a triangle and intersecting the other two sides divides these two sides proportionally.

58*. If a line divides two sides of a triangle proportionally, it is parallel to the third side.

59*. Two triangles are similar if the angles of one are respectively equal to the angles of the other; or if two angles of one are respectively equal to two angles of theother.

60*. Two triangles are similar if their sides are respective-

ly proportional.

61*. Two triangles are similar, if an angle of one equals an angle of the other and the sides including these angles are proportional.

62. The bisector of an interior angle of a triangle divides the opposite side into segments which are proportional to the adjacent sides.

63. The bisector of an exterior angle of a triangle divides the opposite sides externally into segments proportional to the adjacent sides.

64. The segments cut off on two transversals by a series of parallels are proportional.

65. In any right triangle, the perpendicular dropped from the vertex of the right angle to the hypotenuse divides the triangle into two triangles, each similar to the given triangle.

66*. In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

67*. If two chords intersect in a circle, the product of the segments of one is equal to the product of the segments of the other.

68. If from a point outside a circle a secant and a tangent are drawn, the tangent is the mean proportional between the whole secant and its external segment.

69. The perimeters of two similar polygons have the same ratio as any two corresponding sides.

70*. If two polygons are similar, they can be divided into triangles which are similar and similarly placed.

71. If two polygons can be divided into triangles which are similar and similarly placed, the polygons are similar.

72*. The area of a parallelogram is equal to the product of its base by its altitude: $A = bh$.

73. The area of a triangle is equal to half the product of its base by its altitude: $A = 1/2bh$.

74. Two parallelograms, or two triangles, having equal bases or altitudes are equivalent.

75*. The area of a trapezoid is equal to half the product of the base s by its altitude: $A = 1/2(b + b')h$.

76*. The areas of two similar triangles are to each other as the squares of any two corresponding sides.

77*. The areas of two similar polygons are to each other as the squares of any two corresponding sides.

78*. A circle can be circumscribed about any regular polygon.

79. A circle can be inscribed in any regular polygon.

80. If a circle is divided into equal arcs, the chords of these arcs form a regular inscribed polygon, and the tangents at the points of division form a regular circumscribed polygon.

81*. The area of a regular polygon is equal to half the product of its apothem by its perimeter.

82. Regular polygons of the same number of sides are similar.

83. The perimeters of two regular polygons of the same number of sides are to each other as the radii of the inscribed circles, or as the radii of the circumscribed circles.

84. If the number of sides of a regular polygon inscribed in a circle be increased indefinitely, the apothem of the polygon will approach the radius of the circle as its limit.

85. The circumference of a circle is the limit which the perimeters of regular inscribed and circumscribed polygons approach when the number of their sides is increased indefinitely; and the area of the circle is the limit of the areas of these polygons.

86. The circumference of two circles are to each other as their radii.

87. The circumference of a circle is equal to the product of its radius by twice the constant number pi:

$$C = 2 \pi r.$$

88. The area of a circle is equal to half the ~~number~~ product of its circumference by its radius; or to the product of the square of its radius by the constant number pi:

$$A = \pi r^2.$$

89. The areas of two circles are to each other as the squares of their radii.

Constructions

1. To bisect a given line-segment.
2. To bisect a given angle.
3. To draw a perpendicular to a given line through a given point outside the line.
4. At a given point of a given line, to erect a perpendicular to the line.
5. To construct an angle equal to a given angle.
6. Through a given point to draw a straight line parallel to a given straight line.
7. To divide a given line-segment into a given number of equal parts.
8. To construct a triangle, when the three sides are given.
9. To construct a triangle, when two sides and the included angle are given.
10. To construct a triangle, when two angles and a side are given.
11. To circumscribe a circle about a given triangle.
12. To inscribe a circle in a given triangle.
13. To draw a tangent to a given circle through a given

point on the circle.

14. To draw a tangent to a circle through a given external point.

15. To divide a line-segment into parts proportional to given line-segments.

16. To construct a fourth proportional to three given line-segments.

17. To construct the mean proportional between two given line segments.

18. On a given line-segment as side, to construct a polygon similar to a given polygon.

19. To inscribe a regular hexagon in a given circle.

20. To inscribe a square in a given circle.

Note: Occasionally an original will be so framed that a solution will occur more readily to the candidate who is familiar with such important geometrical facts as the properties of the 30° and the 45° right triangles. *

* College Entrance Examination Board Syllabus p. 5.

Appendix C

In analyzing the examinations set by the College Entrance Examination Board it sometimes became necessary to use a theorem in proving an original which is not found in the 1923 syllabus (as shown in appendix B) of this examining board. Such propositions were classified according to their paragraph number in "Plane Geometry and Its Uses." * They are:

Art. 73. If a line has two points each equally distant from the ends of a line-segment, it is the perpendicular bisector of the segment.

Art. 105. If one leg of a right triangle is one-half the hypotenuse, its acute angles are 30° and 60°.

Art. 106. If the acute angles of a right triangle are 30° and 60°, the hypotenuse is twice the shorter leg.

Art. 113. The sum of the exterior angles formed by extending one side at each vertex of a polygon is two straight angles.

Art. 118. A diagonal of a parallelogram divides it into two congruent triangles.

Art. 133. The locus of points equidistant from the sides of a given angle is the bisector of the angle.

Art. 146. Any two oblique line-segments meeting the given line at equal distances from the foot of a perpendicular are equal; and conversely, equal oblique line-segments meet the given line at equal distances from the foot of the perpendicular. and,

The oblique line-segments meeting the given line at unequal distances from the foot of the perpendicular are unequal and the one at the greater distance is the greater; and conversely, unequal oblique line-segments meet the given line at unequal distances from the foot of the perpendicular, the longer at the greater distance.

* Merick, Newell, Harper. Plane Geometry and Its Uses. New York: Row, Peterson and Company, 1929.

Art. 189. The line of centers of two intersecting circles bisects their common chord at right angles.

Art. 199. A central angle is "equal in degrees" to or "is measured by" its intercepted arc.

Art. 232. In any triangle the square of a side opposite an acute angle is equal to the sum of the squares of the other two sides diminished by twice the product of one of these sides and the projection of the other upon it.

Art. 233. In any obtuse triangle the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides increased by twice the product of one of these sides and the projection of the other upon it.

Art. 280E. If in a right triangle a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the perpendicular is a mean proportional between the segments of the hypotenuse.

Art. 310. To inscribe ina circle a polygon of n sides, when n is any of the following numbers: 3, 6, 12, 24, etc.; 4, 8, 16, 32, etc.; 5, 10, 20, 40, etc.; and 15, 30, 60, etc.

Art. 230. If s is the side of an equilateral triangle and h is the altitude, $h = s/2\sqrt{3}$. Area of equilateral triangle,

$$A = s^2/4\sqrt{3}$$